Large Charge for NRCFT

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arXiv:1809.06371, arXiv:2010.07942, arXiv:2111.12094, arXiv:2403.18898, arXiv:2501.10505, arXiv:2510.26876

WHO'S WHO

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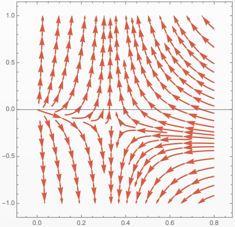
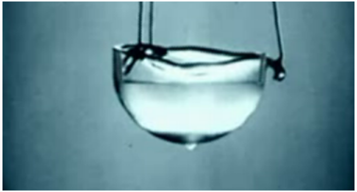
WHY ARE WE HERE? CONFORMAL FIELD THEORIES

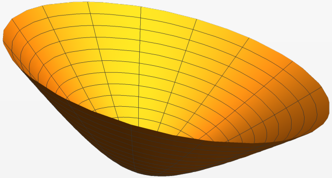
extrema of the RG flow



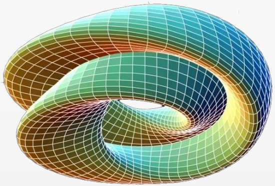
critical phenomena



quantum gravity



string theory



WHY ARE WE HERE? CONFORMAL FIELD THEORIES ARE HARD

Most conformal field theories (CFTs) lack nice limits where they become simple and solvable.

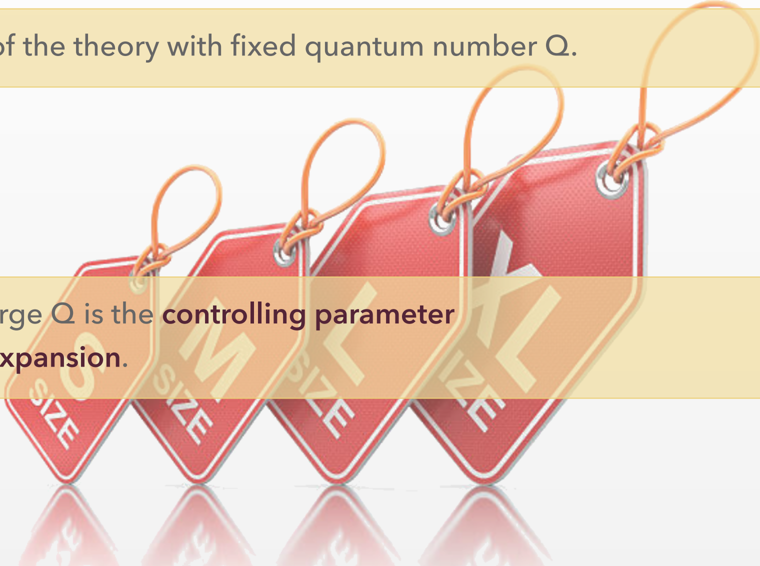
No parameter of the theory can be dialed to a simplifying limit.

WHY ARE WE HERE? CONFORMAL FIELD THEORIES ARE HARD

In presence of a **symmetry** there can be **sectors of the theory** where anomalous dimension and OPE coefficients simplify.

THE IDEA

Study **subsectors** of the theory with fixed quantum number Q.



In each sector, a large Q is the **controlling parameter** in a **perturbative expansion**.

NOT AN ORIGINAL IDEA

NO BOOTSTRAP HERE!

This approach is **orthogonal to bootstrap**.

We will use an effective action.

We will access sectors that are difficult to reach with bootstrap.

(However, arXiv:1710.11161).

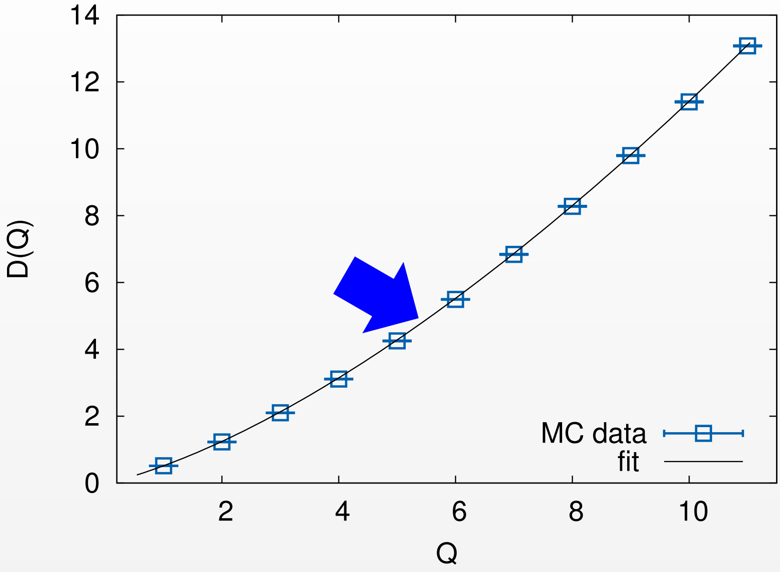
CONCLUSIONS

We consider the $O(N)$ **vector model in three dimensions.** In the IR it flows to a **conformal fixed point** [Wilson & Fisher].

We find an explicit formula for the **dimension of the lowest primary at fixed charge**:

$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi} c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$

CONCLUSIONS: O(2)



SCALES

We want to write a **Wilsonian effective action.**

Choose a cutoff Λ, separate the fields into high and low frequency φ_H, φ_L and do the path integral over the high-frequency part:

$$e^{iS_\Lambda(\varphi_L)} = \int \mathcal{D}\varphi_H \, e^{iS(\varphi_H, \varphi_L)}$$

SCALES

We want to write a **Wilsonian effective action.**

Choose a cutoff Λ, separate the fields into high and low frequency φ_H, φ_L and do the path integral over the high-frequency part:

too hard

$$e^{iS_\Lambda(\varphi_L)} = \int \mathcal{D}\varphi_H \, e^{iS(\varphi_H, \varphi_L)}$$

SCALES

- We look at a finite box of typical **length R**
- The U(1) charge Q fixes a **second scale** $\rho^{1/2} \sim Q^{1/2}/R$

$$\frac{1}{R} \ll \Lambda \ll \rho^{1/2} \sim \frac{Q^{1/2}}{R} \ll \Lambda_{UV}$$

For $\Lambda \ll \rho^{1/2}$ the **effective action is weakly coupled and under perturbative control** in powers of ρ^{-1}.

NON-LINEAR SIGMA MODEL

In a generic theory™, picking the lowest state of fixed charge induces a spontaneous symmetry breaking.
The low-energy physics is described by a **Goldstone field** χ.

Using conformal invariance, the most general action must take the form

$$L[\chi] = k_{3/2} (\partial_\mu \chi \partial^\mu \chi)^{3/2} + k_{1/2} R (\partial_\mu \chi \partial^\mu \chi)^{1/2} + \dots$$

These are the leading terms in the expansion around the classical solution $\chi = \mu t$. All other terms are suppressed by powers of $1/Q$.

NON-LINEAR SIGMA MODEL

The **energy of the lowest state** for this action has the form

$$E = \frac{c_{3/2}}{\sqrt{V}} Q^{3/2} + R c_{1/2} \sqrt{V} Q^{1/2} + \ldots$$

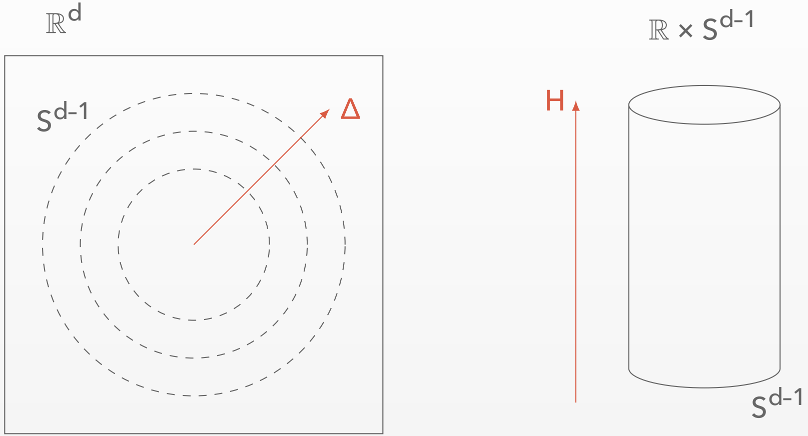
The leading quantum effect is the **Casimir energy of the conformal Goldstone**.

$$E_G = \frac{1}{2\sqrt{2}} \zeta(-\tfrac{1}{2}|S^2) = -0.0937\ldots$$

This is the unique contribution of order Q^0.

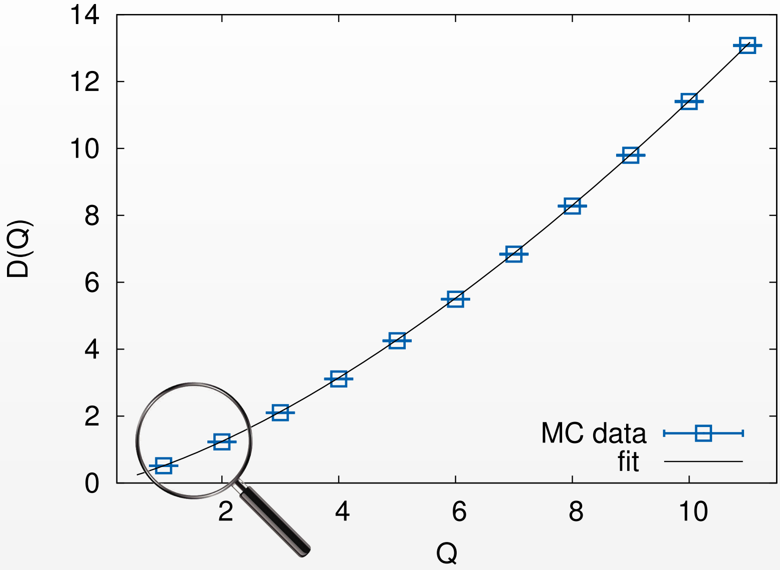
STATE-OPERATOR CORRESPONDENCE

The anomalous dimension on $\mathbb{R}^d$ is the energy in the cylinder frame.



Protected by conformal invariance: a well-defined quantity.

TOO GOOD TO BE TRUE?



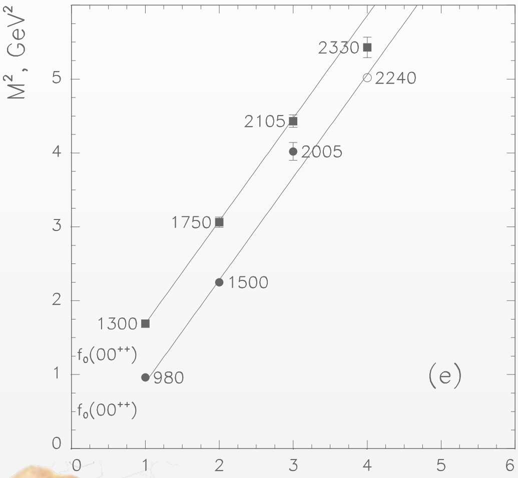
TOO GOOD TO BE TRUE?

Think of **Regge trajectories**.

The prediction of the theory is

$$m^2 \propto J\left(1 + \mathcal{O}\left(J^{-1}\right)\right)$$

but experimentally everything works so well at small J that String Theory was invented.



SELECTED TOPICS IN THE LARGE CHARGE EXPANSION

- O(2) model — [Hellerman, DO, Reffert, Watanabe] [Monin, Pirtskhalava, Rattazzi, Seibold]
- fermions — [Komargodski, Mezei, Pal, Raviv-Moshe] [Antipin, Bersini, Panopoulos] [Hellerman, Dondi, Kalogerakis, Moser, DO, Reffert]
- holography — [Nakayama] [Loukas, DO, Reffert, Sarkar] [de la Fuente] [Guo, Liu, Lu, Pang] [Giombi, Komatsu, Offertaler]
- large N — [Álvarez-Gaumé, DO, Reffert] [Giombi, Hyman]
- ε double-scaling — [Badel, Cuomo, Monin, Rattazzi] [Arias-Tamargo, Rodriguez-Gomez, Russo] [Antipin, Bersini, Sannino, Wang, Zhang] [Jack, Jones]
- non-relativistic CFTs — [Kravec, Pal] [Hellerman, Swanson] [Favrod, DO, Reffert] [DO, Reffert, Pellizzani] [Hellerman, DO, Reffert, Pellizzani, Swanson]
- $\mathcal{N} = 2$ — [Hellerman, Maeda] [Hellerman, Maeda, DO, Reffert, Watanabe] [Bourget, Rodriguez-Gomez, Russo] [Grassi, Komargodski, Tizzano] [Cremonesi, Lanza, Martucci]
- bootstrap — [Jafferis, Zhiboedov]
- resurgence — [Dondi, Kalogerakis, DO, Reffert] [Antipin, Bersini, Sannino, Torres] [Watanabe]

WHAT HAPPENED?

We started from a CFT.
There is no mass gap, there are **no particles**, there is **no Lagrangian**.

We picked a sector.
In this sector the physics is described by a **semiclassical configuration** plus massless fluctuations.

The full theory has no small parameters but we can study this sector with a **simple effective field theory (EFT)**.
We are in a **strongly coupled** regime but we can compute physical observables using **perturbation theory**.

would you like to know more?

TODAY'S TALK

The EFT a non-relativistic conformal field theory

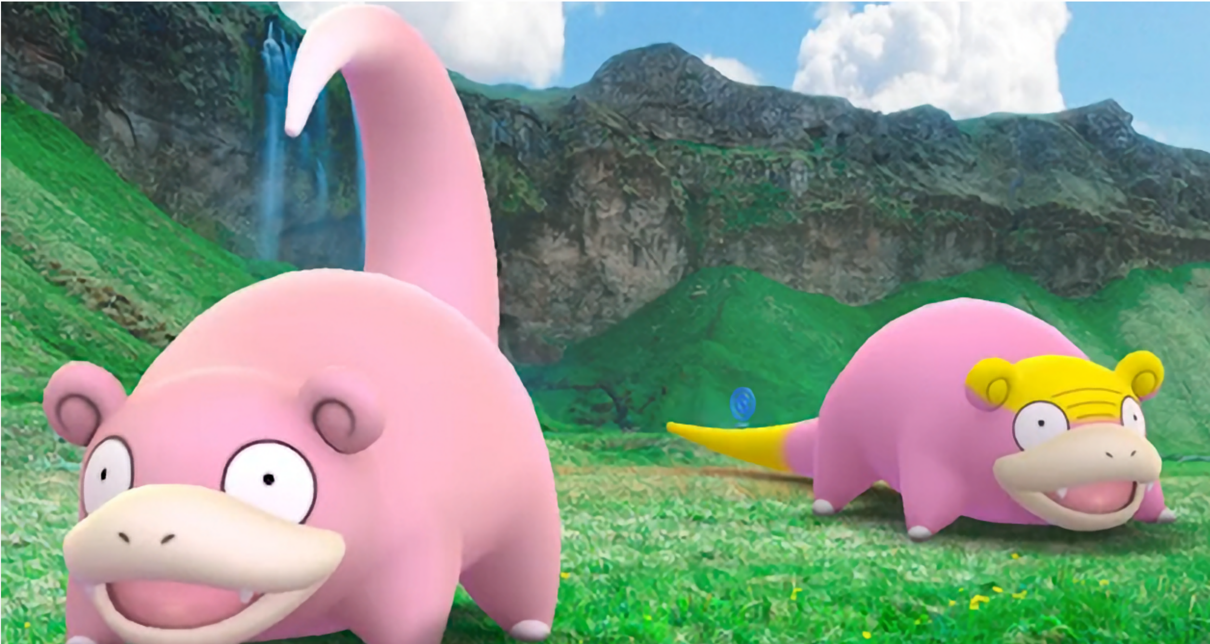
- An EFT for a CFT.
- Experimental realization (ultracold atoms, unnuclear physics).
- Comparison with experiment.

PARENTAL

ADVISORY

EXPLICIT CONTENT

NON-RELATIVISTIC CFT



THE SCHRÖDINGER GROUP

The Schrödinger group describes the symmetries of the **free-particle Schödinger equation**

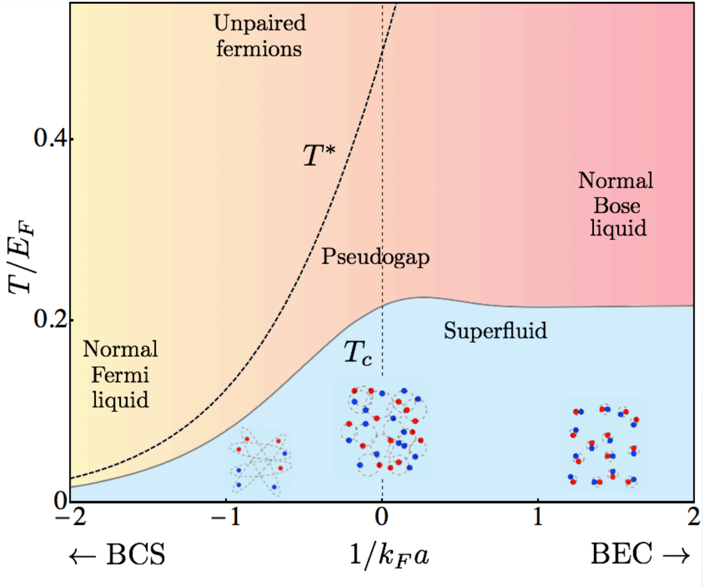
$$i\frac{\partial}{\partial t}\psi = -\frac{\triangle}{2}\psi$$

Its algebra contains the **Galileian algebra**, plus a **central extension** and **two more generators** that, together with time translations form an $SL(2,\mathbb{R})$ algebra.

Contraction of the conformal group, just like the Galileian group is a contraction of the Poincaré group.

It is the symmetry of a **non-relativistic conformal field theory**.

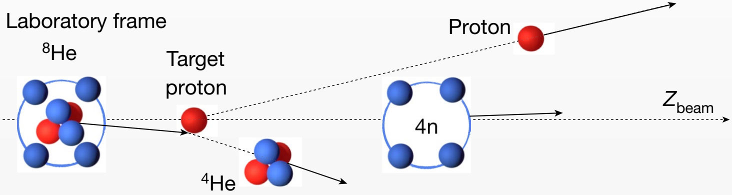
ATOMS AT CRITICALITY

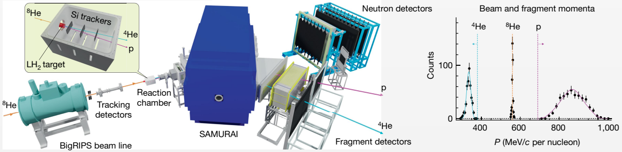


UNNUCLEAR PHYSICS

Neutrons have a **large negative scattering length a ≈ –19 fm** compared to the effective range of the interaction $r_0 \approx 2.8\text{fm}$.

There is a range of energies in which a system of neutrons behaves as a **composite object**, controlled by a NRCFT.





A LARGE-CHARGE EFT: THE INVARIANT OBJECTS

The physics in a sector of fixed charge is described by a **Goldstone field π**.

Start from a **Schrödinger-invariant** object

$$U = \partial_t \chi - \frac{1}{2}\partial_i \chi \partial_i \chi$$

In the fixed-charge vacuum $\chi = \mu t + \pi$, where μ is an increasing function of Q

If we add a background U(1) field A_0

$$U = \partial_t \chi - A_0 - \frac{1}{2}\partial_i \chi \partial_i \chi$$

and a new Schödinger-invariant operator appears:

$$Z = \triangle A_0 - \frac{1}{3}(\triangle \chi)^2$$

Now we want to write the **most general EFT** with these operators.

A LARGE-CHARGE EFT: SCALING DIMENSIONS

The **most general operator** in the EFT is

$$\mathcal{O}^{(n,m)} = (\partial_i U)^{2m} Z^n U^{5/2-(3m+2n)}$$

where m and n are integers to have a local action.

Time and space scale differently. The dynamical exponent is $z = 2$

$$[\partial_i] = 1 \qquad [\partial_t] = z = 2$$

and

$$[U] = 2 \qquad [Z] = 4$$

and all in all,

$$[dt\, d^3x] = -5 \qquad [\mathcal{O}] = 5.$$

A LARGE-CHARGE EFT: Q-DIMENSION

We have an infinite number of operators. We need an ordering principle.
In pion physics it would be the number of derivatives.
Here the system has no intrinsic scales. We use the scale induced by the fixed charge: μ. On the ground state $\chi = \mu t$, so

$$[\mathcal{O}^{(m,n)}]_\mu = 4 - 2(m+n)$$

The leading action is for $m = n = 0$:

$$L = c_0 U^{5/2} = c_0 \left(\partial_t \chi - A_0 - \frac{1}{2}\partial_i \chi \partial_i \chi\right)^{5/2}$$

PHYSICAL OBSERVABLES

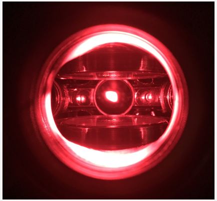
What do people measure?

In the case of **ultracold Fermi gas**, there is a system of atoms confined in a (laser) **harmonic trap**.

The quantities of interest:

- energy of the system
- fluctuation spectrum over the ground state

We need to add a background harmonic potential

$A_0 =$  $= \frac{\omega^2}{2} r^2$

PHYSICAL OBSERVABLES

What do people measure?

In **nuclear experiments**: consider a reaction

$$A_1 + A_2 \to B + \underbrace{n + n + \cdots + n}_{Q}$$

The energy spectrum of B is continuous, with some maximal value E_0 and the cross-section around E_0 has the form

$$\frac{d\sigma}{dE} \sim (E - E_0)^{\Delta(Q)-5/2}$$

where $\Delta(Q)$ is the **conformal dimension** of the unnucleus operator $\mathcal{O}_Q$ that describes the neutrons:

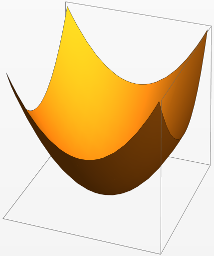
$$\langle \mathcal{O}_Q(t,x)\mathcal{O}_Q(0,0)\rangle \propto \frac{\theta(t)}{t^{\Delta(Q)}} e^{-ix^2/(2t)}$$

STATE-OPERATOR CORRESPONDENCE

State operator correspondence in CFT: operators inserted in $\mathbb{R}^d$ are mapped to states on $\mathbb{R} \times S^{d-1}$. The conformal dimension maps to energy.

In NRCFT there is a similar story.

Operators in $\mathbb{R}^d$ ⇒ states on $\mathbb{R}^d$ with a background harmonic potential.

$A_0 =$ 

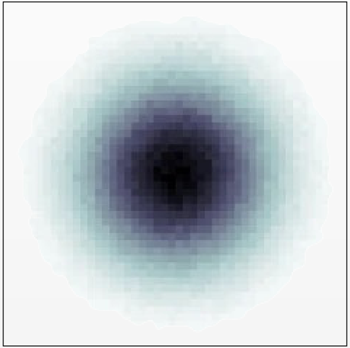
To compute $\Delta(Q)$ we add a background field and compute the energy.
It is the same problem!

INHOMOGENEITY: THE CLOUD

The harmonic trap breaks translation invariance.

The ground state configuration is not homogeneous, but it's a cloud.

The density at the edge goes to zero



The EFT is valid in the bulk, but broken at the edge.

EDGE OPERATORS AND NLO ACTION

We need to add more terms to the EFT. Invariant **quantity localized at the edge**

$$\mathcal{Z}_p = Z^p \delta(U) (\partial_i U)^{(7-4p)/3}$$

this has dimension

$$[\mathcal{Z}_p]_\mu = \frac{5-2p}{3}$$

The first contribution appears at NNLO.

The NLO action is

$$L = c_0 U^{5/2} + c_1 \frac{(\nabla U)^2}{U^{1/2}} + c_2 U^{1/2} \left[(\triangle \chi)^2 - 3 (\nabla \otimes \nabla \chi)^2 \right]$$

where c_0, c_1 and c_2 are Wilson coefficients.
The **controlling parameter is ω/μ**.

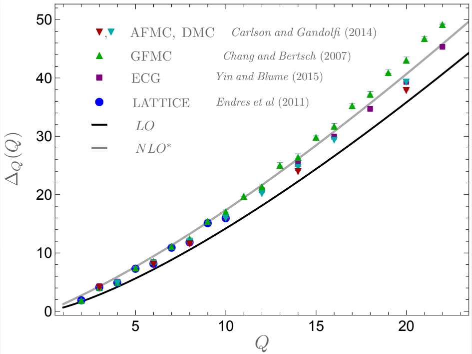
ENERGY OF THE CLOUD

Energy of the ground state

$$\frac{E(Q)}{\omega} = \Delta(Q) = \frac{3^{3/4}}{4}\xi^{1/2}Q^{4/3} - 3^{2/3}\sqrt{2}\pi^2\xi c_1 Q^{2/3} + \mathcal{O}\left(Q^{5/9}\right)$$

ξ is the Bertsch parameter, depending on c_0.

Wilsonian parameters from lattice simulations



$$\xi = 0.372(5)$$

$$c_1 = -0.0537(2)$$

FLUCTUATIONS

The fluctuations about the ground state are described by a Goldstone.

For $A_0 = 0$, the Schrödinger symmetry fixes the dispersion relation

$$q_0 = \sqrt{\frac{2\mu}{3}}q$$

The confining potential changes the story.

- No plane waves
- Discrete spectrum

WHAT TO COMPUTE?

- The response function, i.e. the propagator for the density fluctuations

$$\chi(x_1, x_2; t_1 - t_2) = \langle \delta\rho(x_1, t_1)\delta\rho(x_2, t_2)\rangle_T$$

- Fourier transform on time and relative position $x_1 - x_2$. Integrate over the position of the center of mass $x_1 + x_2$.
  By the **fluctuation-dissipation theorem**, the imaginary part is the dynamic structure factor

$$S(q_0, q) = -\frac{1}{\pi}\mathrm{Im}\left(\chi(q_0, q)\right)$$

For a translation-invariant system

$$S(q_0, q) = \delta\left(q_0 - \sqrt{\frac{2\mu}{3}}q\right)$$

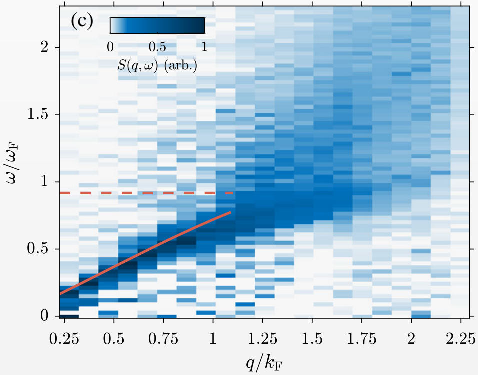
We define the dispersion relation is the curve along which $S(q_0, q)$ is peaked.

DYNAMIC STRUCTURE FACTOR

Alternatively, from **Fermi's Golden Rule**

$$S(q_0, q) = \sum_n |\langle 0|\delta\rho(n)|n\rangle|^2 \delta(q_0 - E(n))$$

Measured experimentally with Bragg spectroscopy



Is it concave or convex?

H. Biss et al, PRL 128, 100401 (2022)

FLUCTUATIONS

Fluctuations at NLO are controlled by the action

$$
\begin{aligned}
L[\pi] = &-\frac{5}{8}c_0\sqrt{\mu - V}\left(2(\mu - V)(\nabla\pi)^2 - 3\dot{\pi}^2\right) \\
&+ c_1\left[\frac{(\nabla\dot{\pi})^2}{\sqrt{\mu - V}} + \frac{\nabla V \cdot \nabla\dot{\pi}^2}{2(\mu - V)^{3/2}} + \frac{(\nabla V)^2\left(2(\mu - V)(\nabla\pi)^2 + 3\dot{\pi}^2\right)}{8(\mu - V)^{5/2}}\right] \\
&+ c_2\sqrt{\mu - V}\left((\Delta\pi)^2 - 3(\nabla \otimes \nabla\pi)^2\right) .
\end{aligned}
$$

To find an approximate solution we use a **WKB expansion**

$$
\pi(t, x) = e^{iq_0 t}\pi(r)Y_{\ell m}(\theta, \varphi),
$$

$$
\pi(u) = \exp\left[\frac{i}{\delta}\mathcal{S}_0(u) + \mathcal{S}_1(u) + i\delta\mathcal{S}_2(u) + \delta^2\mathcal{S}_3(u) + \dots\right].
$$

where δ **is a new small parameter.**

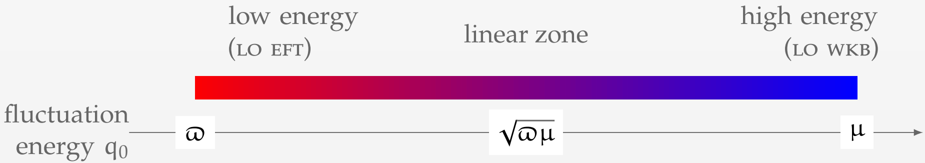
FLUCTUATIONS

- The confining potential introduces a scale ω.
- The charge introduces another scale μ, and then there is the
- Energy of the fluctuations q_0.

Two dimensionless parameters

$$\eta = \frac{q_0}{\mu}$$ controlling the EFT,

$$\delta = \frac{\omega}{q_0}$$ controlling the WKB expansion.



LINEAR REGIME

In the linear regime we use LO EFT and LO WKB (physical optics approximation).

$$\pi(u) = D\frac{\sin\left(\sqrt{3}\frac{q_0}{\omega}\int_0^u \frac{dw}{\sqrt{1-\bar{V}(w)}}\right)}{u\sqrt{1-\bar{V}(u)}}\,.$$

Regularity at the cloud edge $\bar{V}(u) = 1$ gives the quantization condition for q_0 (spectrum)

$$\frac{q_0}{\omega}\sqrt{3}\int_0^1 \frac{dw}{\sqrt{1-\bar{V}(w)}} = n\pi, \qquad n \in \mathbb{Z}.$$

HIGH ENERGY REGIME

NLO EFT and LO WKB

$$\mathcal{S}_0 = \pm\sqrt{3}\int_{u_0}^{u} dw \, \frac{1}{\sqrt{1-\bar{V}(w)}}\left(1+\frac{2}{5c_0}\frac{c_1-3c_2}{\left(1-\bar{V}(w)\right)^2}\frac{q_0^2}{\mu^2}+\mathcal{O}\left(\frac{q_0^4}{\mu^4}\right)\right),$$

$$\mathcal{S}_1 = k_1 - \log\left(u\sqrt{1-\bar{V}}\right) + \frac{c_1-9c_2}{5c_0\,(1-\bar{V})^2}\frac{q_0^2}{\mu^2} + \mathcal{O}\left(\frac{q_0^4}{\mu^4}\right).$$

- $1/\mu^2$ correction
- The edge position changes and with it the spectrum

$$\sqrt{3}\frac{q_0}{\omega}\int_0^{u_1} dw \, \frac{1}{\sqrt{1-\bar{V}(w)}}\left(1+\frac{2}{5c_0}\frac{c_1-3c_2}{(1-\bar{V}(w))^2}\frac{q_0^2}{\mu^2}\right) = n\pi.$$

LOW ENERGY REGIME

LO EFT and NLO WKB

$$\pi(u) = D\,\frac{\sin\left(\frac{q_0}{\omega}\sqrt{3}\int_0^u \frac{dw}{\sqrt{1-\bar{V}(w)}} + \frac{\omega}{q_0}\frac{1}{4\sqrt{3}}\int_0^u \frac{F_\ell[\bar{V}(w)]}{\sqrt{1-\bar{V}(w)}}\,dw\right)}{u\sqrt{\left(1+\frac{\omega^2}{12q_0^2}F_0(\bar{V}(u))\right)(1-\bar{V}(u))}}\,.$$

and the spectrum is

$$\frac{q_0}{\omega}\sqrt{3}\int_0^1 \frac{dw}{\sqrt{1-\bar{V}(w)}} + \frac{\omega}{q_0}\frac{1}{4\sqrt{3}}\int_0^1 \frac{F_\ell[\bar{V}(w)]}{\sqrt{1-\bar{V}(w)}}\,dw = n\pi\,, \qquad n = 1, 2, 3, \ldots$$

where

$$F_\ell[\bar{V}(u)] = \partial_u^2\bar{V}(u) + \frac{3}{u}\,\partial_u\bar{V}(u) - \frac{2\ell(\ell+1)}{u^2}(1-\bar{V}(u))\,.$$

DYNAMIC STRUCTURE FACTOR

$$S(q_0, q) = \sum_n |\langle 0|\delta\rho(n)|n\rangle|^2 \delta(q_0 - E(n))$$

In terms of the field

$$\delta\rho = \frac{15 c_0}{2} \mu^{1/2} \sqrt{1 - \bar{V}} \frac{\partial}{\partial t} \pi(t, u)$$

In the physical optics approximation

$$\langle 0|\delta\rho(\mathbf{q})|n\rangle \propto \frac{\mu^{1/4}\sqrt{c_0 q_0}}{q} \int_0^1 du \sin\left(q R_{cl} u\right) \sin\left(\frac{\mathcal{S}_0(u)}{\delta}\right) .$$

We solve the integral with a saddle point approximation for $\delta \ll 1$

HIGH ENERGY REGIME

The saddle equation is

$$q R_{cl} = \frac{1}{\delta} \partial_u \mathcal{S}_0(\bar{u})$$

and we can approximate

$$S(q_0, \mathbf{q}) \approx \mu^{1/2} \frac{c_0}{\omega q^2} \sum_n \frac{q_0^2}{\left|\partial_u^2 \mathcal{S}_0(\bar{u})\right|} \sin^2(q R_{cl} \bar{u}) \delta(q_0 - \frac{n\pi\omega}{\mathcal{S}_0(1)})\Big|_{q=\frac{q_0}{\sqrt{2\mu}} \partial_u \mathcal{S}_0(\bar{u})} .$$

At leading order, the saddle condition is

$$q = \frac{q_0}{\sqrt{2\mu}} \frac{\sqrt{3}}{\sqrt{1 - \bar{V}(u)}} = \sqrt{\frac{3}{2(\mu - V(r))}} q_0 .$$

This reproduces the local density approximation in condensed matter physics.

DISPERSION RELATION

The dispersion relation is the curve along which $S(q_0, q)$ has a peak.

$$S(q_0, \mathbf{q}) \approx \mu^{1/2} \frac{c_0}{\omega q^2} \sum_n \frac{q_0^2}{\left|\partial_u^2 \mathcal{S}_0(\bar{u})\right|} \sin^2(q R_{cl} \bar{u}) \delta\left(q_0 - \frac{n\pi\omega}{\mathcal{S}_0(1)}\right)\bigg|_{q = \frac{q_0}{\sqrt{2\mu}} \partial_u \mathcal{S}_0(\bar{u})} .$$

Plugging in the WKB result

$$\partial_u^2 \mathcal{S}_0 = -\frac{\sqrt{3}\, \partial_u \bar{V}}{2(1 - \bar{V})} = 0$$

the peaks correspond to the stationary points of V (the center of the trap).

DISPERSION RELATION

Plug everything in.
The saddle-point equation is

$$q = \frac{q_0}{\sqrt{2\mu}} \mathcal{S}_0(0) = \sqrt{\frac{3}{2\mu}} q_0 \left(1 + \frac{2}{5c_0}(c_1 - 3c_2)\frac{q_0^2}{\mu^2} + \mathcal{O}(\frac{q_0}{\mu})^4\right),$$

and the dynamic structure factor is peaked along the curve

$$q_0 = \sqrt{\frac{2\mu}{3}} q \left(1 - \frac{4}{15}\frac{c_1 - 3c_2}{c_0}\frac{q^2}{\mu}\right),$$

In the high energy regime the concavity does not depend on the potential but only on the Wilson parameters.

LOW ENERGY REGIME

The expectation value of the density fluctuations localizes around the curve

$$q = \sqrt{\frac{3}{2(\mu - V)}} q_0 \left[1 + \frac{1}{6q_0^2} \left(\partial_r^2 V(r) + 3 \frac{\partial_r V(r)}{r} \right) \right].$$

The peaks of the dynamic structure factor appear at the roots of the equation

$$\partial_r \left[\sqrt{\frac{3}{2(\mu - V)}} q_0 \left[1 + \frac{1}{6q_0^2} \left(\partial_r^2 V(r) + 3 \frac{\partial_r V(r)}{r} \right) \right] \right] = 0.$$

Here the shape of the potential is crucial

LOW ENERGY REGIME

For the harmonic potential

$$q_0 = \sqrt{\frac{2\mu}{3}}q\left(1 - \frac{\omega^2}{q^2\mu} + \dots\right)$$

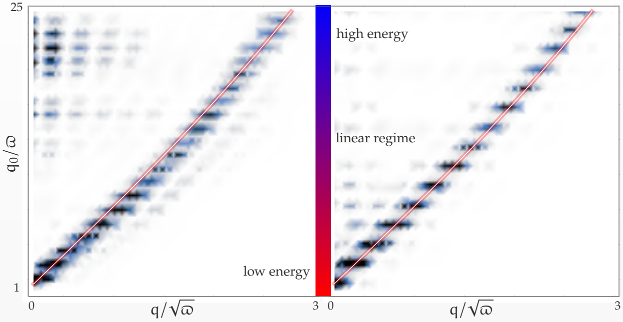
the leading correction is concave.

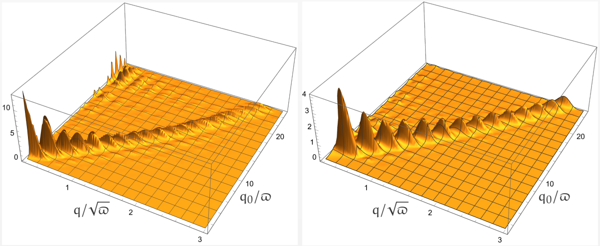
For a superharmonic potential $V = r^{2k}$, the correction becomes smaller

$$q_0 = \sqrt{\frac{2\mu}{3}}q\left(1 + \mathcal{O}\left(\frac{\omega^{2k}}{q^{2k}\mu^k}\right)\right)$$

Was to be expected: the flatter the potential, the closer we get to the flat box where there are no $1/q$ corrections.

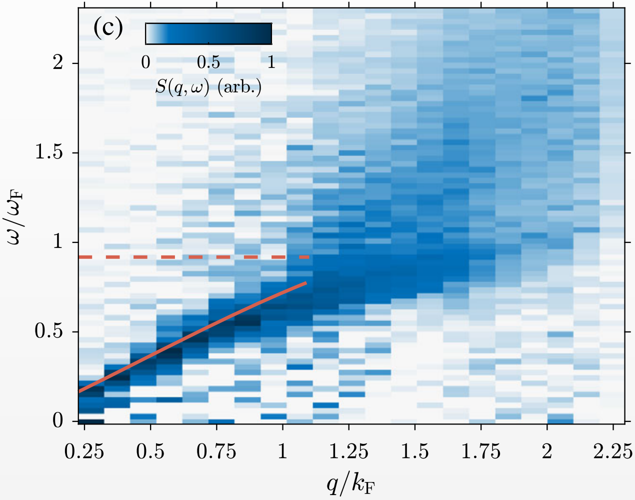
FINAL RESULT





- Numerical evaluation of the integral for $V = r^2$ and $V = r^{16}$.
- The red line is the saddle
- Discrete spectrum
- The flatter the potential, the more peaked the function

BACK TO THE EXPERIMENT



- Experimental measure on the Fermi gas for $V \approx r^{16}$.
- Seems concave in the low-energy regime.

CONCLUSIONS

- With the large-charge approach we can study **strongly-coupled systems perturbatively**.
- Select a sector and we write a **controllable effective theory**.
- The strongly-coupled physics is (for the most part) subsumed in a **semiclassical state**.
- Precise and **testable predictions**.
- Remarkable agreement with **lattice** and **experiment**.