Fermionic CFTs at large charge and large N

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arXiv:1505.01537, arXiv:1610.04495, arXiv:1909.02571, arXiv:2008.03308, arXiv: 2211.15318 ...

WHO'S WHO

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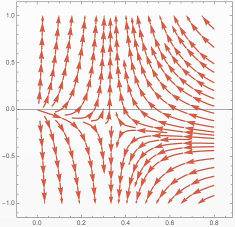
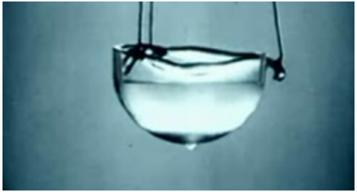
I. Swanson;
M. Watanabe (Amsterdam).

WHY ARE WE HERE? CONFORMAL FIELD THEORIES

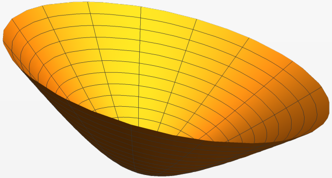
extrema of the RG flow



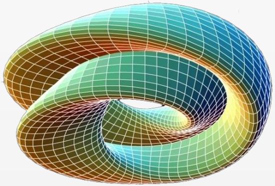
critical phenomena



quantum gravity



string theory



WHY ARE WE HERE? CONFORMAL FIELD THEORIES ARE HARD

Most conformal field theories (CFTs) lack nice limits where they become simple and solvable.

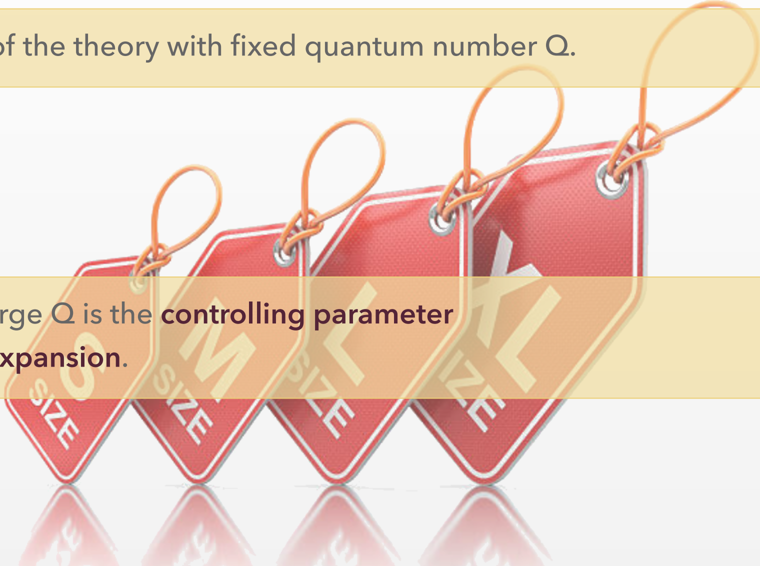
No parameter of the theory can be dialed to a simplifying limit.

WHY ARE WE HERE? CONFORMAL FIELD THEORIES ARE HARD

In presence of a **symmetry** there can be **sectors of the theory** where anomalous dimension and OPE coefficients simplify.

THE IDEA

Study **subsectors** of the theory with fixed quantum number Q.



In each sector, a large Q is the **controlling parameter** in a **perturbative expansion**.

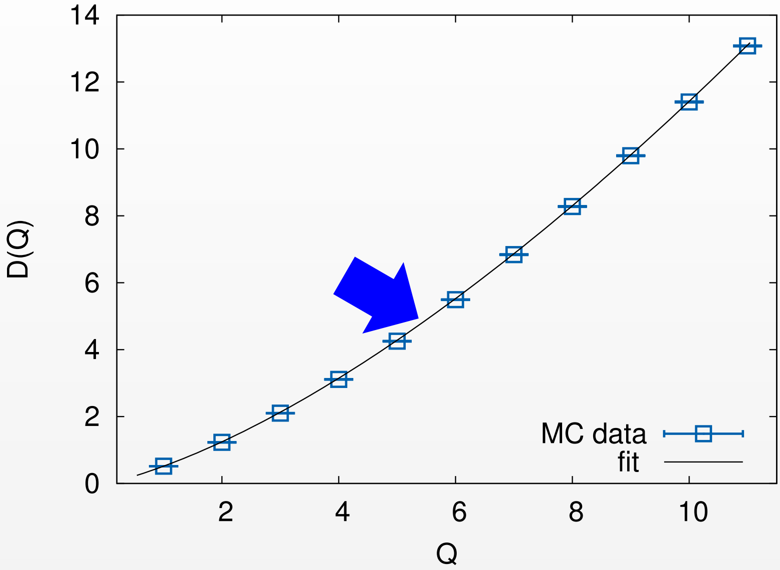
CONCRETE RESULTS

We consider the $O(N)$ **vector model in three dimensions**. In the IR it flows to a **conformal fixed point** [Wilson & Fisher].

We find an explicit formula for the **dimension of the lowest primary at fixed charge**:

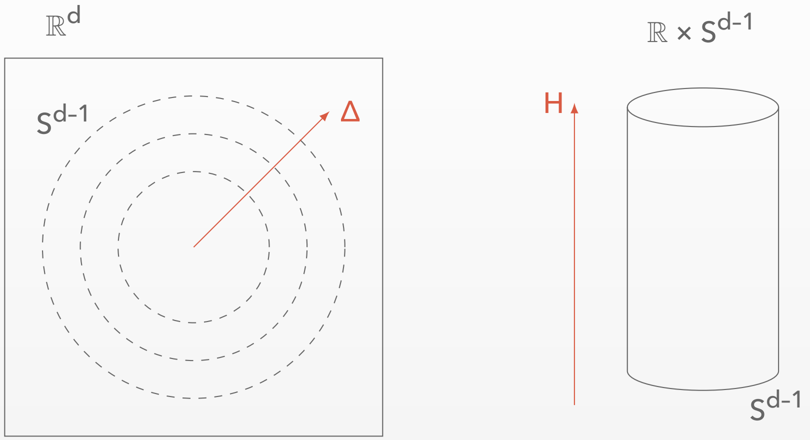
$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi} c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$

SUMMARY OF THE RESULTS: O(2)



STATE-OPERATOR CORRESPONDENCE

The anomalous dimension on $\mathbb{R}^d$ is the energy in the cylinder frame.



Protected by conformal invariance: a well-defined quantity.

SCALES

We want to write a **Wilsonian effective action.**

Choose a cutoff Λ, separate the fields into high and low frequency φ_H, φ_L and do the path integral over the high-frequency part:

$$e^{iS_\Lambda(\varphi_L)} = \int \mathcal{D}\varphi_H \, e^{iS(\varphi_H, \varphi_L)}$$

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SCALES

- We look at a finite box of typical **length R**
- The U(1) charge Q fixes a **second scale $\rho^{1/2} \sim Q^{1/2}/R$**

$$\frac{1}{R} \ll \Lambda \ll \rho^{1/2} \sim \frac{Q^{1/2}}{R} \ll \Lambda_{UV}$$

For $\Lambda \ll \rho^{1/2}$ the **effective action is weakly coupled and under perturbative control** in powers of ρ^{-1}.

NON-LINEAR SIGMA MODEL

In a generic theory™, picking the lowest state of fixed charge induces a spontaneous symmetry breaking.
The low-energy physics is described by a **Goldstone field** χ.

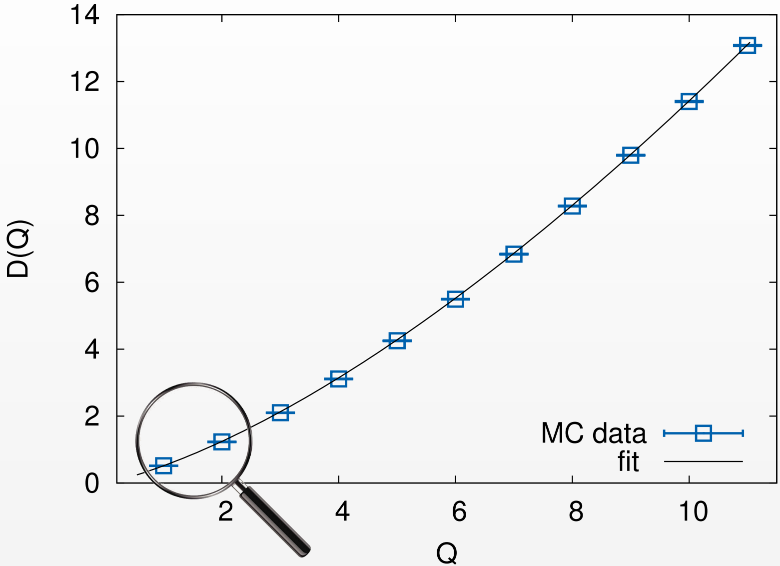
Using conformal invariance, the most general action must take the form

$$L[\chi] = k_{3/2}(\partial_\mu \chi \partial^\mu \chi)^{3/2} + k_{1/2} R(\partial_\mu \chi \partial^\mu \chi)^{1/2} + \dots$$

These are the leading terms in the expansion around the classical solution $\chi = \mu t$. All other terms are suppressed by powers of $1/Q$.

The **energy of the lowest state** for this action is the **conformal dimension of the lowest operator** of given charge Q.

TOO GOOD TO BE TRUE?



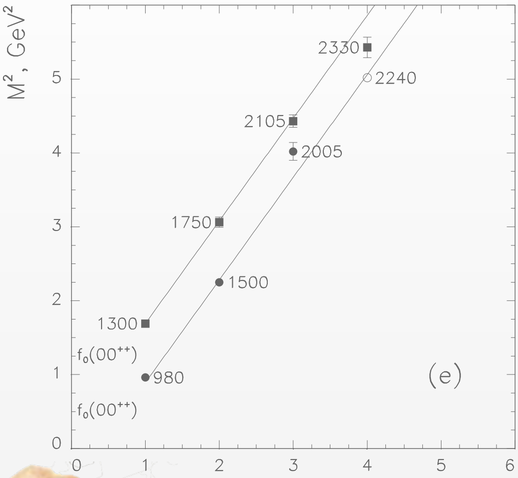
TOO GOOD TO BE TRUE?

Think of **Regge trajectories**.

The prediction of the theory is

$$m^2 \propto J\left(1 + \mathcal{O}\left(J^{-1}\right)\right)$$

but experimentally everything works so well at small J that String Theory was invented.



TOO GOOD TO BE TRUE?

The unreasonable effectiveness

of the **large charge expansion**.

SELECTED TOPICS IN THE LARGE CHARGE EXPANSION

- **O(2) model** [Hellerman, DO, Reffert, Watanabe] [Monin, Pirtskhalava, Rattazzi, Seibold]
- **fermions** [Komargodski, Mezei, Pal, Raviv-Moshe] [Antipin, Bersini, Panopoulos] [Hellerman, Dondi, Kalogerakis, Moser, DO, Reffert]
- **holography** [Loukas, DO, Reffert, Sarkar] [de la Fuente] [Guo, Liu, Lu, Pang] [Giombi, Komatsu, Offertaler]
- **large N** [Álvarez-Gaumé, DO, Reffert] [Giombi, Hyman]
- **ε double-scaling** [Badel, Cuomo, Monin, Rattazzi] [Arias-Tamargo, Rodriguez-Gomez, Russo] [Antipin, Bersini, Sannino, Wang, Zhang] [Jack, Jones]
- **non-relativistic CFTs** [Kravec, Pal] [Helllerman, Swanson] [Favrod, DO, Reffert] [DO, Reffert, Pellizzani] [Hellerman, DO, Reffert, Pellizzani, Swanson]
- **$\mathcal{N} = 2$** [Hellerman, Maeda] [Hellerman, Maeda, DO, Reffert, Watanabe] [Bourget, Rodriguez-Gomez, Russo] [Grassi, Komargodski, Tizzano] [Cremonesi, Lanza, Martucci]
- **bootstrap** [Jafferis, Zhiboedov]
- **resurgence** [Dondi, Kalogerakis, DO, Reffert] [Antipin, Bersini, Sannino, Torres] [Watanabe]

WHAT HAPPENED?

We started from a CFT.
There is no mass gap, there are **no particles**, there is **no Lagrangian**.

We picked a sector.
In this sector the physics is described by a **semiclassical configuration** plus massless fluctuations.

The full theory has no small parameters but we can study this sector with a **simple effective field theory (EFT)**.
We are in a **strongly coupled** regime but we can compute physical observables using **perturbation theory**.

TODAY'S TALK

The EFT for the $O(2)$ model in $2+1$ dimensions

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Justify and prove all my claims from first principles in four-Fermi models

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The EFT for the $O(2)$ model in $2+1$ dimensions

- An EFT for a CFT.
- The physics at the saddle.
- State/operator correspondence for anomalous dimensions.

Justify and prove all my claims from first principles in four-Fermi models

TODAY'S TALK

The EFT for the $O(2)$ model in 2 + 1 dimensions

Justify and prove all my claims from first principles in four-Fermi models

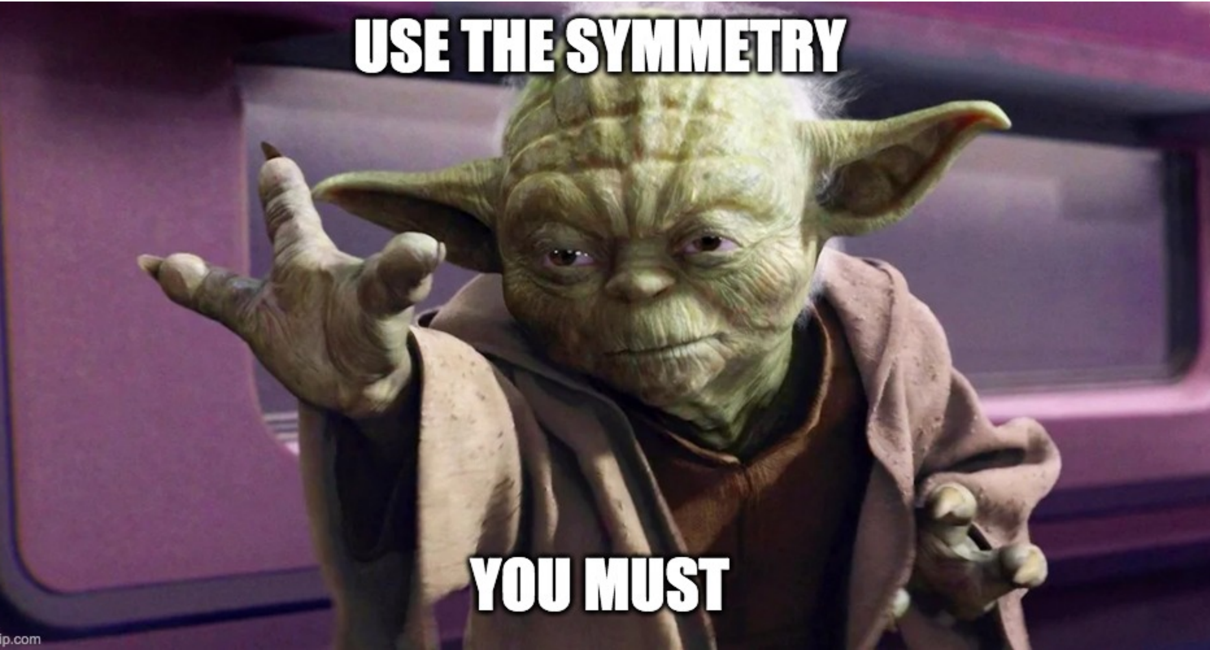
- well-defined asymptotic expansion (in the technical sense)
- justify why the expansion works at small charge
- compute the coefficients in the effective action in large-N

PARENTAL

ADVISORY

EXPLICIT CONTENT

AN EFT FOR A CFT



THE O(2) MODEL

The simplest example is the Wilson-Fisher (WF) point of the O(2) model in three dimensions.

- Non-trivial fixed point of the φ^4 action

$$L_{UV} = \partial_\mu \varphi^* \partial_\mu \varphi - u(\varphi^* \varphi)^2$$

- Strongly coupled
- In nature: ^{4}He.
- Simplest example of spontaneous symmetry breaking.
- **Not accessible** in perturbation theory. **Not accessible** in $4 - \varepsilon$. **Not accessible** in large N.
- Lattice. Bootstrap.

CHARGE FIXING

We consider a **subsector of fixed charge Q**.

Generically, the classical solution at fixed charge **breaks spontaneously $U(1) \to \emptyset$**.

We have one **Goldstone boson** χ.

AN ACTION FOR χ

Start with two derivatives:

$$L[\chi] = \frac{f_\pi}{2} \partial_\mu \chi \partial_\mu \chi - C^3$$

(χ is a Goldstone so it is dimensionless.)

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We want to describe a CFT: we can **dress with a dilaton**

$$L[\sigma, \chi] = \frac{f_\pi e^{-2f\sigma}}{2} \partial_\mu \chi \partial_\mu \chi - e^{-6f\sigma} C^3 + \frac{e^{-2f\sigma}}{2} \left(\partial_\mu \sigma \partial_\mu \sigma - \frac{\xi R}{f^2} \right)$$

The fluctuations of χ give the Goldstone for the broken $U(1)$, the fluctuations of σ give the (massive) Goldstone for the broken conformal invariance.

LINEAR SIGMA MODEL

We can put together the two fields as

$$\Sigma = \sigma + i f_\pi \chi$$

and rewrite the action in terms of a complex scalar

$$\varphi = \frac{1}{\sqrt{2f}} e^{-f\Sigma}$$

We get

$$L[\varphi] = \partial_\mu \varphi^* \partial^\mu \varphi - \xi R \varphi^* \varphi - u(\varphi^* \varphi)^3$$

Only depends on dimensionless quantities $b = f^2 f_\pi$ and $u = 3(Cf^2)^3$.

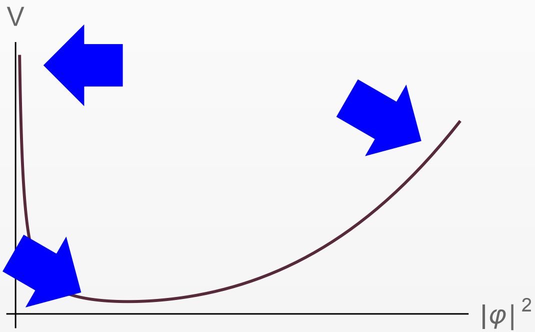
Scale invariance is manifest.

The field φ is some complicated function of the original φ.

CENTRIFUGAL BARRIER

The $O(2)$ symmetry acts as a shift on χ.

Fixing the charge is the same as adding a **centrifugal term** $\propto \dfrac{1}{|\varphi|^2}$.



GROUND STATE

We can find a fixed-charge solution of the type

$$\chi(t, x) = \mu t \qquad \sigma(t, x) = \frac{1}{f} \log(v) = \text{const.},$$

where

$$\mu \propto Q^{1/2} + \ldots \qquad v \propto \frac{1}{Q^{1/2}}$$

The classical energy is

$$E = c_{3/2} V Q^{3/2} + c_{1/2} R V Q^{1/2} + \mathcal{O}\left(Q^{-1/2}\right)$$

FLUCTUATIONS

The fluctuations over this ground state are described by two modes.

- A universal "**conformal Goldstone**". It comes from the breaking of the $U(1)$.

$$\omega = \frac{1}{\sqrt{2}} p$$

- The **massive dilaton**. It controls the magnitude of the quantum fluctuations.
  All quantum effects are controled by $1/Q$.

$$\omega = 2\mu + \frac{p^2}{2\mu}$$

  (This is a heavy fluctuation around the semiclassical state. It has nothing to do with a light dilaton in the full theory)

NON-LINEAR SIGMA MODEL

Since σ is heavy we can integrate it out and write a non-linear sigma model (NLSM) for χ alone.

$$L[\chi] = k_{3/2}(\partial_\mu \chi \partial^\mu \chi)^{3/2} + k_{1/2} R(\partial_\mu \chi \partial^\mu \chi)^{1/2} + \dots$$

These are the leading terms in the expansion around the classical solution $\chi = \mu t$.
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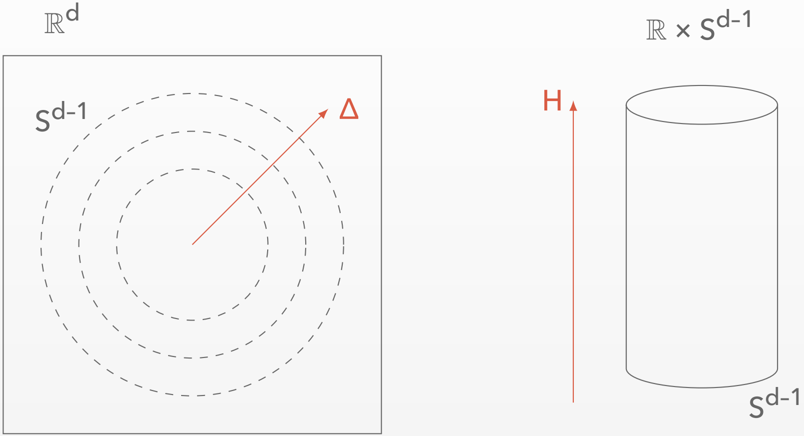

Protected by conformal invariance: a well-defined quantity.

CONFORMAL DIMENSIONS

We know the energy of the ground state.

The leading quantum effect is the **Casimir energy of the conformal Goldstone**.

$$E_G = \frac{1}{2\sqrt{2}}\zeta(-\tfrac{1}{2}|S^2) = -0.0937\ldots$$

This is the unique contribution of order Q^0.

Final result: the **conformal dimension of the lowest operator of charge Q** in the O(2) model has the form

$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}}Q^{3/2} + 2\sqrt{\pi}c_{1/2}Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$

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LARGE N VS. LARGE CHARGE

GROSS—NEVEU AND NAMBU—JONA—LASINIO

Simplest four-Fermi models

$$S_{GN}[\psi] = \int dt\, d\Sigma \sum_{i=1}^{N} \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - \frac{g}{2N}(\bar{\psi}\psi)^2$$

$$S_{NJL}[\psi, \varphi] = \int dt\, d\Sigma \sum_{i=1}^{N} \bar{\psi}_i \gamma^\mu \partial_\mu \psi_i - \frac{g}{N}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5 \psi)^2\right]$$

Respectively $O(4N)$ and $U(N) \times U(1)$ symmetry.

Non-trivial CFT in the ultraviolet (UV).

Convenient to introduce collective fields

$$\sigma = \frac{g}{N}\bar{\psi}\psi$$

$$\varphi = \frac{g}{N}\left(\bar{\psi}\psi + \bar{\psi}\gamma^5\psi\right)$$

GNY AND NJL MODELS AT LARGE N

$$S_{GNY}[\psi,\sigma] = \int dt\, d\Sigma \sum_{i=1}^{N} \bar{\psi}_i \left(\gamma^\mu \partial_\mu + \sigma\right) \psi_i + \frac{1}{2g_Y} \partial_\mu \sigma \partial_\mu \sigma.$$

$$S_{NJL}[\psi,\varphi] = \int dt\, d\Sigma \sum_{i=1}^{N} \bar{\psi}_i \left[\gamma^\mu \partial_\mu + \varphi \left(\frac{1+\gamma_5}{2}\right) + \varphi^* \left(\frac{1-\gamma_5}{2}\right)\right] \psi_i + \frac{1}{g_Y} \partial_\mu \varphi^* \partial_\mu \varphi$$

They flow in the IR to the same CFTs.

We want to fix $U(1)$ charges.

$$\psi_i \to e^{i\alpha} \psi_i,$$

$$\psi \to e^{i\alpha\gamma_5} \psi, \qquad \varphi \to e^{-2i\alpha} \varphi,$$

and compute the partition function at fixed charge

$$Z(Q) = \mathrm{Tr}\left[e^{-\beta H} \delta(\hat{Q} - Q)\right].$$

FIX THE CHARGE

$$Z_\Sigma(Q) = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{i\theta Q}\, \mathrm{Tr}\left[e^{-\beta H} e^{-i\theta \hat{Q}}\right].$$

At large N the integral over θ becomes a Legendre transform

$$\Delta(Q) = -\frac{1}{\beta} \log\left(Z_{S^2}(Q)\right) = \sup_{i\theta}\left(i\theta Q - S_{eff}(\theta)\right)$$

The trace is written as a path integral in two ways:

$$Z_\Sigma(Q) = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} \int_{\substack{\psi(2\pi\beta) = -e^{-\theta}\psi(0) \\ \varphi(2\pi\beta) = e^{2i\theta}\varphi(0)}} \mathcal{D}\varphi_i\, e^{-S[\varphi]}$$

$$= \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} \int_{\substack{\psi(2\pi\beta) = -\psi(0) \\ \varphi(2\pi\beta) = \varphi(0)}} \mathcal{D}\varphi_i\, e^{-S^{\theta}[\varphi]}$$

EFFECTIVE ACTION: COVARIANT DERIVATIVE

The actions are quadratic in the fermions. We can integrate them out. For the bosonic fields, we expand as vacuum expectation value (VEV) plus fluctuations

$$\sigma = \sigma_0 + \frac{1}{\sqrt{N}}\hat{\sigma}$$

$$\varphi_0 = \varphi_0 + \frac{1}{\sqrt{N}}\hat{\varphi}$$

The leading contribution comes from the VEV:

$$\Omega = S_{\text{eff}} = -N\,\mathrm{Tr}\log\left(\gamma^{\mu}\partial_{\mu} - i\frac{\theta}{\beta}\gamma_3 + \sigma_0\right)$$

$$\Omega = S_{\text{eff}} = -N\,\mathrm{Tr}\log\left(\gamma^{\mu}\partial_{\mu} - i\frac{\theta}{\beta}\gamma_3\gamma_5 + \varphi_0\left(\frac{1+\gamma_5}{2}\right) + \varphi_0^{*}\left(\frac{1-\gamma_5}{2}\right)\right)$$

This has to be minimized with respect to σ_0 and φ_0 (gap equation).

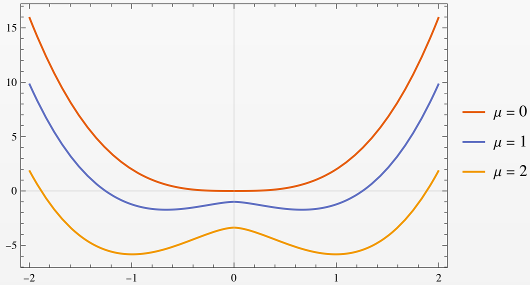
We can read $i\theta/\beta = \mu$ as a chemical potential.

THE NAMBU–JONA–LASINIO MODEL

In the $\beta \to \infty$ limit, on the torus $\Sigma = T^2$, the grand potential is

$$\frac{\Omega}{N} = -\int \frac{d^2p}{(2\pi)^2}\left[\sqrt{(|p|+\mu)^2+|\varphi_0|^2}+\sqrt{(|p|-\mu)^2+|\varphi_0|^2}\right]$$

$$= -\frac{1}{6\pi}\left[3|\varphi|^2\mu\,\mathrm{arctanh}\,\frac{\mu}{\sqrt{|\varphi|^2+\mu^2}}+(\mu^2-2|\varphi|^2)\sqrt{|\varphi|^2+\mu^2}\right]$$



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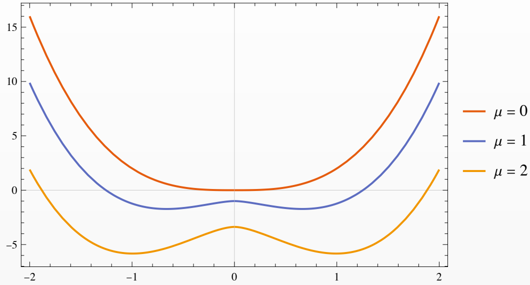


The gap equation admits a non-vanishing solution for any value of μ

$$\varphi_0 = \mu\sqrt{\kappa_0^2 - 1} = 0.6627\cdots\times\mu$$

This is the physics that we had discussed before: fixing the charge induces a spontaneous symmetry breaking.

The field φ is the order parameter for the superfluid phase transition.

COOPER PAIRS

The scalar φ can be understood as a composite field

$$\varphi = \bar{\psi}\psi + \bar{\psi}\gamma^5\psi$$

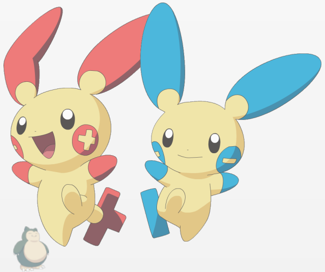
Its meaning is even more transparent after a Pauli-Gürsey transformation:

$$\psi \mapsto \frac{1}{2}\left[(1-\gamma_5)\psi - (1+\gamma_5)C\bar{\psi}^T\right], \qquad \bar{\psi} \mapsto \frac{1}{2}\left[\bar{\psi}(1+\gamma_5) - \psi^T C(1-\gamma_5)\right].$$

because then we identify φ with a Cooper pair

$$\varphi = \psi^t C\psi$$

In presence of an attractive interactions, fermion form pairs that behave as bosons and undergo a Bose-Einstein transition.



CONFORMAL DIMENSIONS

Now we can repeat the computation on S^2.

By the state-operator correspondence, the free energy is the conformal dimension of the lowest operator.

$$\Omega = -\frac{N}{4\pi r_0^2} \sum_{j=1/2}^{\infty} (2j+1)\left(\Omega_+ + \Omega_-\right), \qquad \Omega_\pm = |\varphi|^2 + (\omega_j \pm \mu)^2,$$

where $\omega_j = j + 1/2$ are the eigenvalues on the sphere.

We need to minimize with respect to φ and Legendre transform from the chemical potential μ to the charge Q.

Two regimes:

- The large-charge regime $Q \gg N$,
- The small-charge regime $Q \ll N$

LARGE CHARGE

We need to regularize the sums (ask). The grand potential has two pieces:

$$\Omega_r = -\frac{2N}{3}(r_0\mu)^3\left(3(\kappa_0^2-1)\operatorname{arccoth}\kappa_0 + 3\kappa_0 - 2\kappa_0^3 + 2(\kappa_0^2-1)^{3/2}\right) + \dots,$$

$$\Omega_d = N\left(\frac{4(\varphi_0 r_0)^3}{3} + \frac{\varphi_0 r_0}{3} - \frac{1}{60\varphi_0 r_0} + \dots\right).$$

In the large-charge regime we look for a solution of the form

$$\varphi_0 r_0 = \sqrt{\kappa_0^2 - 1}\left(\mu r_0 + \frac{\kappa_1}{\mu r_0} + \frac{\kappa_2}{(\mu r_0)^3} + \dots\right).$$

After some algebra we can solve the gap equation order-by-order

$$\kappa_0 \tanh\kappa_0 = 1, \qquad \kappa_1 = -\frac{1}{12\kappa_0^2}, \qquad \kappa_2 = \frac{33 - 16\kappa_0^2}{1440\kappa_0^6}, \qquad \dots$$

ORDER N

$$F_{S^2}(Q) = \frac{4N}{3}\left(\frac{Q}{2N\kappa_0}\right)^{3/2} + \frac{N}{3}\left(\frac{Q}{2N\kappa_0}\right)^{1/2} - \frac{11-6\kappa_0^2}{360\kappa_0^2}\left(\frac{Q}{2N\kappa_0}\right)^{-1/2} + \mathcal{O}\left(e^{-\sqrt{Q/(2N)}}\right)$$

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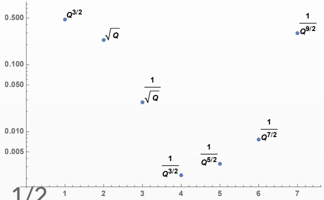

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SMALL CHARGE

In the small charge regime, we need again to separate the regular from the divergent part:

$$\Omega_r = -2N \sum_{\ell=1} \ell \Bigg[\sqrt{(\ell + \tfrac{1}{2} + \hat{\mu} r_0)^2 + (\varphi_0 r_0)^2} + \sqrt{(\ell - \tfrac{1}{2} - \hat{\mu} r_0)^2 + (\varphi_0 r_0)^2}$$

$$- \sqrt{(\ell + \tfrac{1}{2})^2 + (\varphi_0 r_0)^2} + \sqrt{(\ell - \tfrac{1}{2})^2 + (\varphi_0 r_0)^2} \Bigg],$$

$$\Omega_d = -4N \sum_{\ell=1}^{\infty} (\ell + \tfrac{1}{2}) \sqrt{(\ell + \tfrac{1}{2})^2 + (\varphi_0 r_0)^2}.$$

The divergent part can be understood in terms of zeta functions (ask)

SMALL CHARGE

Now we expand around $\mu = 1/(2r_0)$ in powers of μ.
This is the conformal coupling to a cylinder in three dimensions.

$$\mu = 1/(2r_0) + \mu_2 \varphi_0^2 r_0 + \mu_4 \varphi_0^4 r_0^3 + \dots.$$

With some algebra

$$\frac{\Delta(Q)}{2N} = \frac{1}{2}\left(\frac{Q}{2N}\right) + \frac{2}{\pi^2}\left(\frac{Q}{2N}\right)^2 + \dots$$

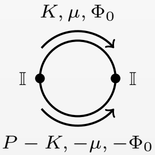
Consistent with the fact that φ has charge two and dimension 1/2, so that $\Delta(Q) \approx Q/2$.

ORDER N^0

In the EFT we had a universal term. Is it really universal?
Here it can only appear at order 1/N, so we need to compute the fluctuations around the vacuum. The fermion propagator is

$$D_\psi(P) = (-i\not{P} + \varphi_0 - \mu\Gamma_3\Gamma_5)^{-1}.$$

The fluctuations for the scalars are obtained with a one-loop fermion computation. For example the real part of φ interacts with itself via



$$= D^{-1}(P) = -\int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}\left[D_\psi(K) D_\psi(P-K)\right],$$

ORDER N°: THE UNIVERSAL GOLDSTONE

The final result for the two real components is

$$D^{-1}(P) = \begin{pmatrix} \frac{\kappa_0 \mu}{\pi} + \frac{2\kappa_0^2\left(2\kappa_0^2-1\right)\omega^2+\left(3\kappa_0^6-2\kappa_0^4-2\kappa_0^2+2\right)p^2}{24\pi\kappa_0^3(\kappa_0^2-1)\mu} & -\frac{\kappa_0}{2\pi}\omega \\ \frac{\kappa_0}{2\pi}\omega & \frac{2\kappa_0\omega^2+\kappa_0^3 p^2}{8\pi(\kappa_0^2-1)\mu} \end{pmatrix} + \mathcal{O}(P^3/\mu^3),$$

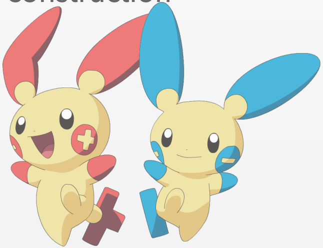
and from here we read the dispersion relations for a massive and for the universal Goldstone mode

$$\omega_1^2 = \frac{1}{2}p^2 + \ldots,$$

$$\omega_2^2 = 12\kappa_0^4 \frac{\kappa_0^2 - 1}{2\kappa_0^2 - 1}\mu^2 + \frac{5\kappa_0^6 - 5\kappa_0^4 - \kappa_0^2 + 2}{2\kappa_0^2(2\kappa_0^2 - 1)}p^2 + \ldots$$

WHAT HAS HAPPENED?

- We have taken a model in which the fixed point is under large-N control
- Fixing the charge results in **spontaneous symmetry breaking**
- The order parameter is a composite field (**Cooper pair**)
- We compute the conformal dimension of the lowest operator of charge Q
- We find a result in **total agreement** with the general EFT construction



THE GROSS–NEVEU–YUKAWA MODEL

Now for the Gross-Neveu-Yukawa model.

Integrating out the Matsubara frequencies we find

$$\frac{\Omega}{N} = -2 \int \frac{d^2 p}{(2\pi)^2} \left\{ \omega_p + \frac{1}{\beta} \log\left(1 + e^{-\beta(\omega_p + \mu)}\right) + (\mu \leftrightarrow -\mu) \right\},$$

with $\omega_p^2 = p^2 + \sigma_0^2$. The gap equation is

$$\sigma_0 - \frac{1}{\beta} \log\left((1 + e^{\beta(\sigma_0 + \mu)})(1 + e^{\beta(\sigma_0 - \mu)}) \right) = 0$$

and admits only the solution $\sigma_0 = 0$.

In other words, in the large-N limit the symmetry is never broken.

This is different from the superfluid EFT behavior.

THE GROSS–NEVEU–YUKAWA MODEL

We can repeat the computation on the sphere. The grand potential is

$$\frac{\Omega}{N} = -\frac{1}{2\pi r_0^2}\left[\sum_{\omega_j > \mu}(2j+1)\omega_j + \mu \sum_{\omega_j < \mu}(2j+1)\right]$$

the solution to the gap equation and the Legendre transform are

$$\frac{Q}{N} = \frac{1}{2\pi r_0^2}\lfloor \mu r_0 \rfloor(\lfloor \mu r_0 \rfloor + 1), \qquad \frac{E}{N} = \frac{1}{6\pi r_0^3}\lfloor \mu r_0 \rfloor(\lfloor \mu r_0 \rfloor + 1)(2\lfloor \mu r_0 \rfloor + 1).$$

This is the physics of a Fermi sphere.

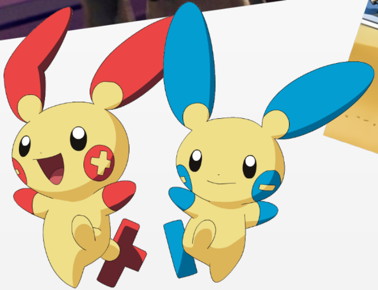
However, the behavior of the conformal dimension is still the same

$$\Delta = \frac{2}{3}\left(\frac{Q}{2N}\right)^{3/2} + \frac{1}{12}\left(\frac{Q}{2N}\right)^{1/2} - \frac{1}{192}\left(\frac{Q}{2N}\right)^{-1/2} + \dots$$

WHAT HAS HAPPENED?

- We have taken a model in which the fixed point is under large-N control
- Fixing the charge **does not result** in spontaneous symmetry breaking
- The physics is the one of a **Fermi sphere**
- The dimension of the lowest operator still obeys the same law
- **Conundrum:** is this a large-N effect or is there a finite-N transition?

CONCLUSIONS



CONCLUSIONS

- With the large-charge approach we can study **strongly-coupled systems perturbatively**.
- Select a sector and we write a **controllable effective theory**.
- The strongly-coupled physics is (for the most part) subsumed in a **semiclassical state**.
- Precise and **testable predictions**.
- **Fermionic** systems at large N
- Spontaneous symmetry breaking vs Fermi sphere.