A large charge to rule strong coupling

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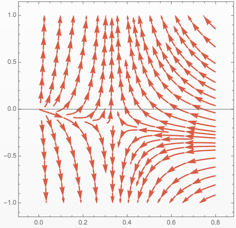
arXiv:1505.01537, arXiv:1610.04495, arXiv:1707.00711, arXiv:1804.01535, arXiv:1902.09542, arXiv:1905.00026,arXiv:1909.02571, arXiv:1909.08642
and more to come…

Who's who

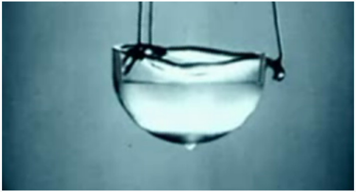
S. Reffert (AEC Bern);
L. Alvarez Gaumé (CERN and SCGP);
F. Sannino (CP3-Origins);
D. Banerjee (DESY);
S. Chandrasekharan (Duke);
S. Hellerman (IPMU);
M. Watanabe (Weizmann).

Why are we here? Conformal field theories

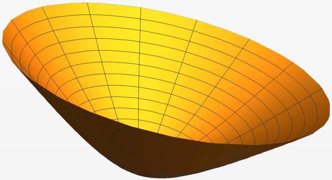
extrema of the RG flow



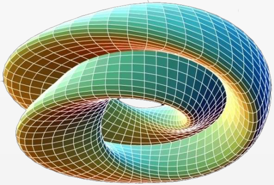
critical phenomena



quantum gravity



string theory



But conformal field theories are hard

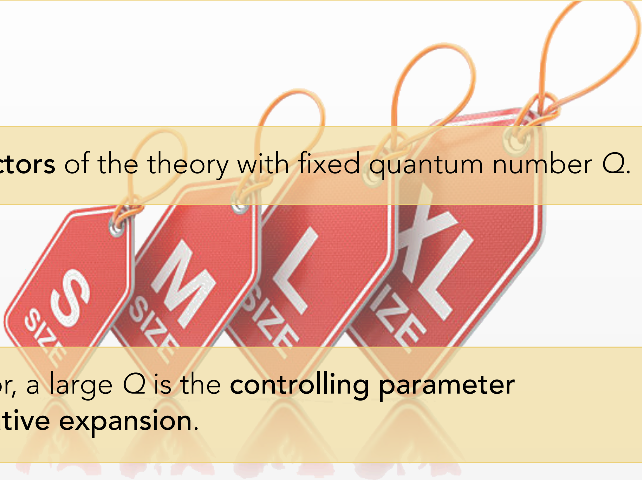
Most conformal field theories (CFTs) lack nice limits where they become simple and solvable.

No parameter of the theory can be dialed to a simplifying limit.

Why are we here? Conformal field theories are hard

In presence of a symmetry there can be sectors of the theory where anomalous dimension and OPE coefficients simplify.

The idea



Study **subsectors** of the theory with fixed quantum number Q.

In each sector, a large Q is the **controlling parameter** in a **perturbative expansion**.

no bootstrap here!

This approach is orthogonal to bootstrap.
We will use an effective action.
We will access sectors that are difficult to reach with bootstrap.
(However, arXiv:1710.11161).

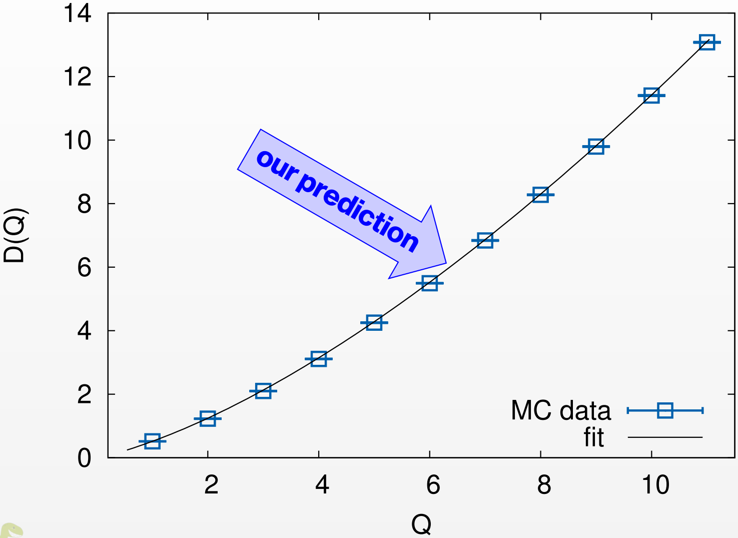
Concrete results

We consider the $O(N)$ vector model in three dimensions. In the IR it flows to a conformal fixed point Wilson & Fisher.

We find an explicit formula for the dimension of the lowest primary at fixed charge:

$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$

Summary of the results: O(2)



Scales

We want to write a Wilsonian effective action.

Choose a cutoff Λ, separate the fields into high and low frequency ϕ_H, ϕ_L and do the path integral over the high-frequency part:

$$e^{iS_\Lambda(\phi_L)} = \int \mathscr{D}\phi_H \, e^{iS(\phi_H,\phi_L)}$$

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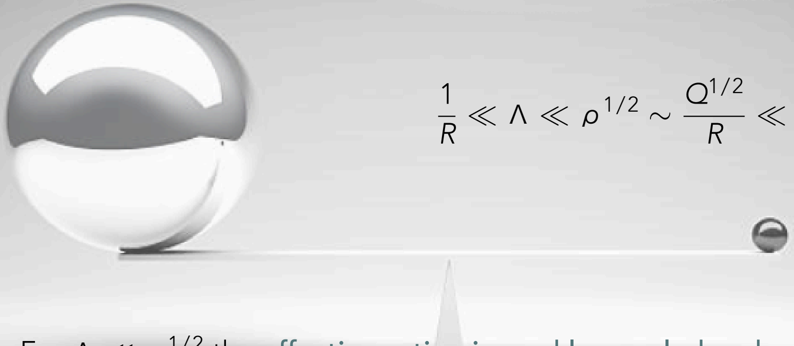
too hard

Scales

- We look at a finite box of typical length R
- The $U(1)$ charge Q fixes a second scale $\rho^{1/2} \sim Q^{1/2}/R$



$$\frac{1}{R} \ll \Lambda \ll \rho^{1/2} \sim \frac{Q^{1/2}}{R} \ll \Lambda_{UV}$$

For $\Lambda \ll \rho^{1/2}$ the effective action is weakly coupled and under perturbative control in powers of ρ^{-1}.

Wilsonian action

The Wilsonian action is fundamentally useless because it contains infinite terms.

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- *maybe* a convergent expansion in derivatives.

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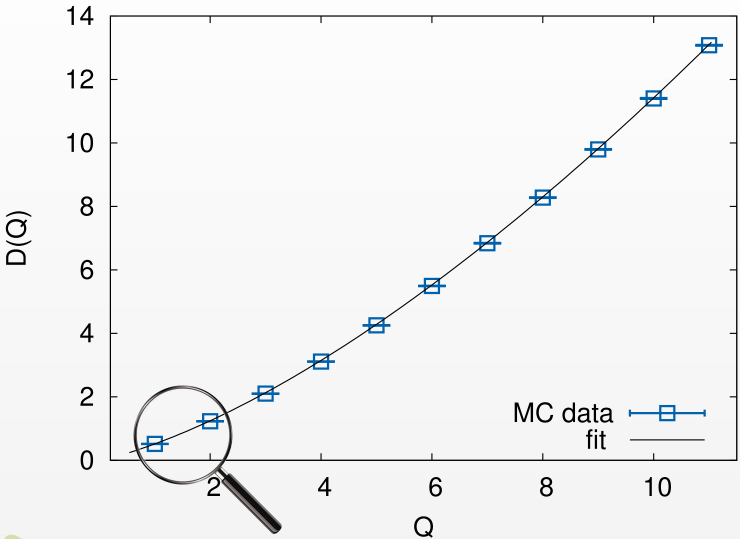
superstition

Too good to be true?

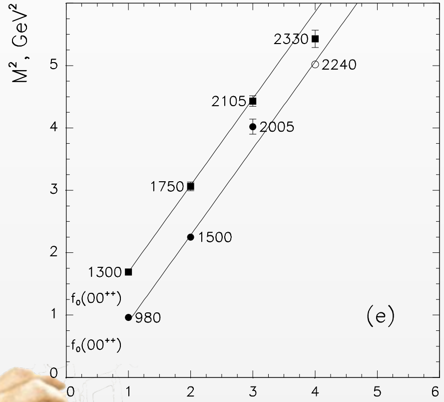


Too good to be true?

Think of Regge trajectories.
The prediction of the theory is

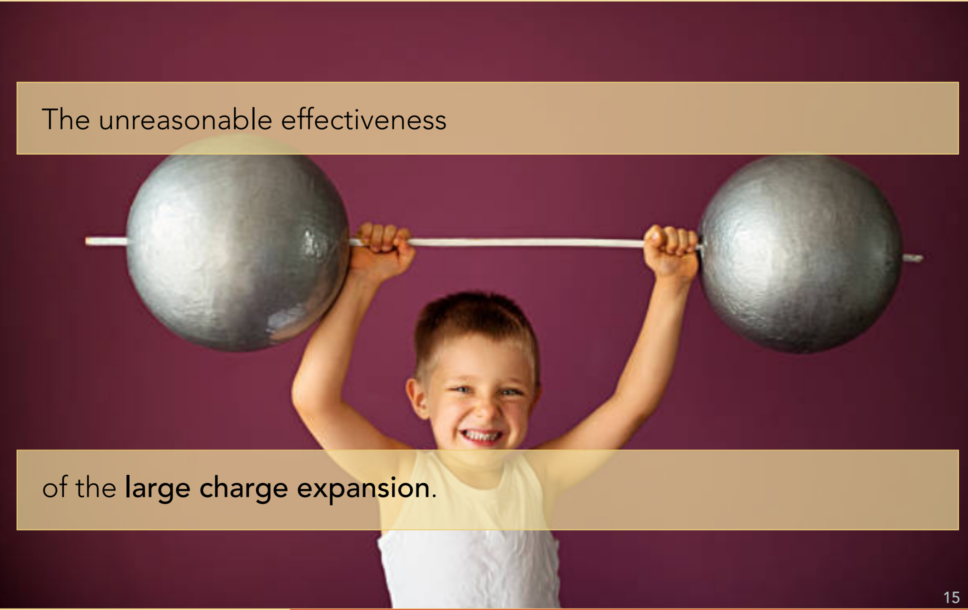
$$m^2 \propto J\left(1 + \mathcal{O}\left(J^{-1}\right)\right)$$

but *experimentally* everything works so well at small J that String Theory was invented.



Too good to be true?

The unreasonable effectiveness



of the **large charge expansion**.

Today's talk

The effective field theory (EFT) for the $O(2)$ model in $d = 3$

- An EFT for a CFT.
- The physics at the saddle.
- State/operator correspondence for anomalous dimensions.

An asymptotically safe theory at large charge

- A QCD-like theory
- Conformal dimensions
- Decoupling

A nearly critical theory at large charge

PARENTAL

ADVISORY

EXPLICIT CONTENT

An EFT for a CFT

The $O(2)$ model

The simplest example is the Wilson–Fisher (WF) point of the $O(2)$ model in three dimensions.

- Non-trivial fixed point of the ϕ^4 action

$$L_{UV} = \partial_\mu \phi^* \partial_\mu \phi - u(\phi^* \phi)^2$$

- Strongly coupled
- In nature: ^4He.
- Simplest example of spontaneous symmetry breaking.
- Not accessible in perturbation theory. Not accessible in $4 - \varepsilon$. Not accessible in large N.
- Lattice. Bootstrap.

Charge fixing

We assume that the $O(2)$ symmetry is not accidental.

We consider a subsector of fixed charge Q.
Generically, fixing the charge breaks it.
It will look like a spontaneous breaking $U(1) \to \emptyset$.

We have one Goldstone boson χ.

An action for χ

Start with two derivatives:

$$L[\chi] = \frac{f_\pi}{2} \partial_\mu \chi \, \partial_\mu \chi - C^3$$

(χ is a Goldstone so it is dimensionless.)

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We want to describe a CFT: we can dress with a dilaton

$$L[\sigma, \chi] = \frac{f_\pi e^{-2f\sigma}}{2} \partial_\mu \chi \, \partial_\mu \chi - e^{-6f\sigma} C^3 + \frac{e^{-2f\sigma}}{2} \left(\partial_\mu \sigma \, \partial_\mu \sigma - \frac{\xi R}{f^2} \right)$$

The fluctuations of χ give the Goldstone for the broken $U(1)$, the fluctuations of σ give the (massive) Goldstone for the broken conformal invariance.

Linear sigma model

We can put together the two fields as

$$\Sigma = \sigma + i f_\pi \chi$$

and rewrite the action in terms of a complex scalar

$$\varphi = \frac{1}{\sqrt{2} f} e^{-f\Sigma}$$

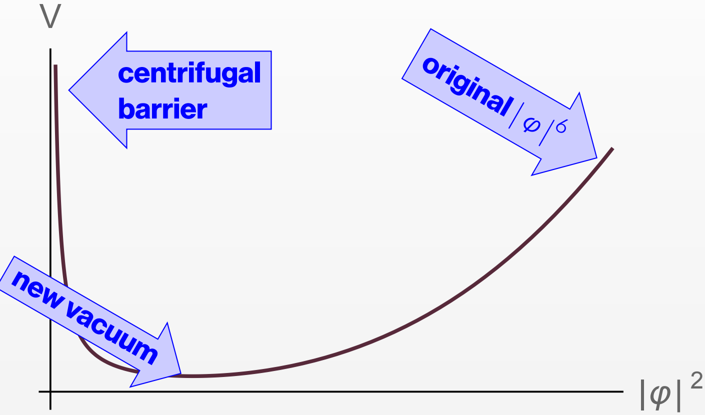
We get

$$L[\varphi] = \partial_\mu \varphi^* \partial^\mu \varphi - \xi R \varphi^* \varphi - u(\varphi^* \varphi)^3$$

Only depends on dimensionless quantities $b = f^2 f_\pi$ and $u = 3(Cf^2)^3$.
Scale invariance is manifest.
The field φ is some complicated function of the original ϕ.

Centrifugal barrier

The $O(2)$ symmetry acts as a shift on χ.
Fixing the charge is the same as adding a centrifugal term $\propto \frac{1}{|\varphi|^2}$.



Ground state

We can find a fixed-charge solution of the type

$$\chi(t,x) = \mu t \qquad \sigma(t,x) = \frac{1}{f}\log(v) = \text{const.},$$

where

$$\mu \propto Q^{1/2} + \dots \qquad v \propto \frac{1}{Q^{1/2}}$$

The classical energy is

$$E = c_{3/2} V Q^{3/2} + c_{1/2} R V Q^{1/2} + \mathcal{O}\left(Q^{-1/2}\right)$$

Fluctuations

The fluctuations over this ground state are described by two modes.

- A universal "conformal Goldstone". It comes from the breaking of the $U(1)$.

$$\omega = \frac{1}{\sqrt{2}} p$$

- The massive dilaton. It controls the magnitude of the quantum fluctuations. All quantum effects are controled by $1/Q$.

$$\omega = 2\mu + \frac{p^2}{2\mu}$$

(This is a heavy fluctuation around the semiclassical state. It has nothing to do with a light dilaton in the full theory)

Non-linear sigma model

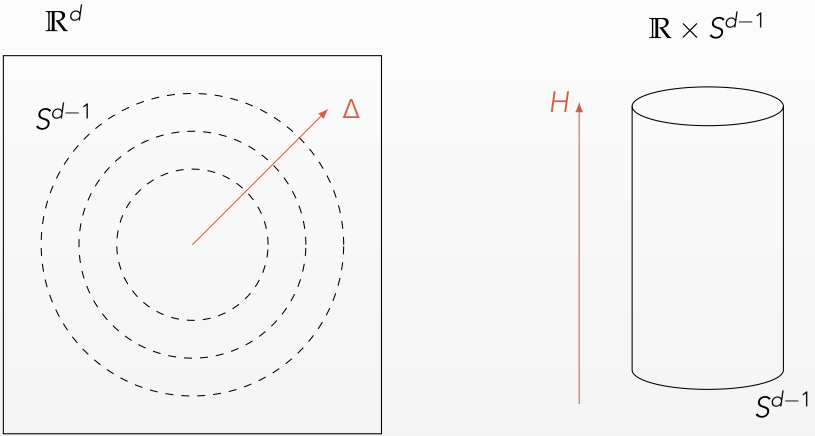
Since σ is heavy we can integrate it out and write a non-linear sigma model (NLSM) for χ alone.

$$L[\chi] = k_{3/2}(\partial_\mu \chi \, \partial^\mu \chi)^{3/2} + k_{1/2} R (\partial_\mu \chi \, \partial^\mu \chi)^{1/2} + \dots$$

These are the leading terms in the expansion around the classical solution $\chi = \mu t$. All other terms are suppressed by powers of $1/Q$.

State-operator correspondence

The anomalous dimension on $\mathbb{R}^d$ is the energy in the cylinder frame.



Protected by conformal invariance: a well-defined quantity.

Conformal dimensions

We know the energy of the ground state.
The leading quantum effect is the Casimir energy of the conformal Goldstone.

$$E_G = \frac{1}{2\sqrt{2}} \zeta\left(-\tfrac{1}{2} | S^2\right) = -0.0937\ldots$$

This is the unique contribution of order Q^0.

Final result: the conformal dimension of the lowest operator of charge Q in the $O(2)$ model has the form

$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$

The $O(2N)$ model

Next step: $O(2N)$.
N charges can be fixed.

Again, homogeneous ground state.
The ground-state energy only depends on the sum of the charges

$$Q = Q_1 + \cdots + Q_N$$

and takes the same form

$$E = \frac{c_{3/2}(N)}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2}(N) Q^{1/2} + \mathcal{O}\left(Q^{-1/2}\right)$$

The coefficients depend on N and cannot be computed in the EFT (but *e.g.* in large-N).

Fluctuations

The symmetry breaking pattern is

$$O(2N) \xrightarrow{\text{exp.}} U(N) \xrightarrow{\text{spont.}} U(N-1)$$

and there are $\dim(U(N)/U(N-1)) = 2N-1$ degrees of freedom (DOF).

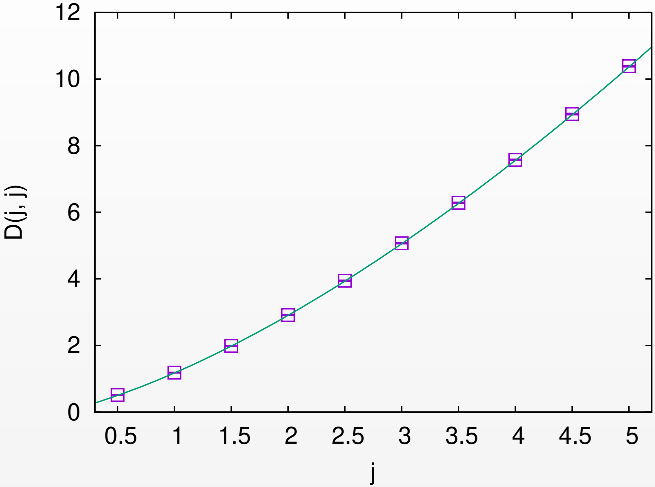
- One singlet, the universal conformal Goldstone $\omega = \frac{1}{\sqrt{2}}p$
- One vector of $U(N-1)$, with quadratic dispersion $\omega = \frac{p^2}{2\mu}$

Each type-II Goldstone counts for two DOF:

$$1 + 2 \times (N-1) = 2N - 1.$$

Only the type-I has a Q^0 contribution: it is universal.

$O(4)$ on the lattice



$$\Delta_j = \frac{c_{3/2}}{2\sqrt{\pi}}(2j)^{3/2} + 2\sqrt{\pi}\, c_{1/2}(2j)^{1/2} - 0.094 + \mathcal{O}\left(j^{-1/2}\right)$$

What happened?

We started from a CFT.
There is no mass gap, there are no particles, there is no Lagrangian.

We picked a sector.

In this sector the physics is described by a semiclassical configuration plus massless fluctuations.

The full theory has no small parameters but we can study this sector with a simple EFT.
We are in a strongly coupled regime but we can compute physical observables using perturbation theory.

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Bridging perturbative and nonperturbative physics

An asymptotically safe QFT

IR vs. UV

We have discussed an infrared (IR) fixed point.
The fixed charge induces a scale $\Lambda_Q = \frac{Q^{1/d}}{r}$.
We need a hierarchy for the scale Λ of the EFT

$$\frac{1}{r} \ll \Lambda \ll \Lambda_Q \ll \Lambda_{UV}$$

The situation improves if we consider a ultraviolet (UV) fixed point.

$$\frac{1}{r} \ll \Lambda_{UV} \ll \Lambda \ll \Lambda_Q$$

and we can take the charge as large as we like.

An asymptotically safe theory

$$\mathcal{L} = -\frac{1}{2}\operatorname{Tr}(F^{\mu\nu}F_{\mu\nu}) + \operatorname{Tr}\left(\bar{Q} i \not{D} Q\right) + y \operatorname{Tr}\left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\right)$$
$$+ \operatorname{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) - u \operatorname{Tr}\left(H^\dagger H\right)^2 - v(\operatorname{Tr} H^\dagger H)^2 - \frac{R}{6}\operatorname{Tr}\left(H^\dagger H\right).$$

In the Veneziano limit of $N_F \to \infty$, $N_C \to \infty$ with the ratio N_F/N_C fixed, this theory is asymptotically safe.

Perturbatively-controlled UV fixed point

$$\alpha_g^* = \frac{26}{57}\varepsilon, \quad \alpha_y^* = \frac{4}{19}\varepsilon, \quad \alpha_h^* = \frac{\sqrt{23}-1}{19}\varepsilon, \quad \alpha_v^* = -0.13\,\varepsilon.$$

An asymptotically safe theory

New features from our point of view

- H is a matrix. There is a large non-Abelian global symmetry
- there are fermions
- there are gluons
- it's a four-dimensional system
- we have a trustable effective action

The scalar sector

The $SU(N_F) \times SU(N_F)$ symmetry is generated by the currents

$$J_L = i\,\mathrm{d}H\,H^\dagger, \qquad J_R = -iH^\dagger\,\mathrm{d}H,$$

and we will be looking for solutions of the classical equations of motion (EOM) at fixed values of the corresponding conserved charges

$$\mathcal{Q}_L = \int \mathrm{d}^3x\,J_L^0, \qquad \mathcal{Q}_R = \int \mathrm{d}^3x\,J_R^0.$$

more precisely

$$\mathrm{spec}(\mathcal{Q}_L) = \{J_1^L, J_2^L, \dots, J_{N_F}^L\}$$
$$\mathrm{spec}(\mathcal{Q}_R) = \{J_1^R, J_2^R, \dots, J_{N_F}^R\}.$$

The scalar sector

Inspired by the $O(2)$ model we use a homogeneous ansatz

$$H_0 = e^{2iMt} B,$$

and the EOM reduce to

$$2M^2 = uB^2 + v\,\mathrm{Tr}\left(B^2\right) - \frac{R}{12}.$$

For simplicity

$$\mathcal{Q}_L = -\mathcal{Q}_R = J \left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right),$$

where $\mathbb{1}$ is the $N_F/2 \times N_F/2$ identity matrix.
The ground state is

$$M = \mu \left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right), \qquad B = b \left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & \mathbb{1} \end{array}\right).$$

Ground state energy and fluctuations

The ground state has energy

$$E = \frac{3}{2}\frac{N_F^2}{\alpha_h + \alpha_v}\left(\frac{2\pi}{V}\right)^{1/3}\left[\mathcal{J}^{4/3} + \frac{R}{36}\left(\frac{V}{2\pi^2}\right)^{2/3}\mathcal{J}^{2/3} - \frac{1}{144}\left(\frac{R}{6}\right)^2\left(\frac{V}{2\pi^2}\right)^{4/3}\mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right)\right]$$

which is a natural expansion in

$$\mathcal{J} = 2J\frac{\alpha_h + \alpha_v}{N_F} \gg 1$$

We have again an expansion in powers of the charge.
The leading exponent is $4/3$ because we are in four dimensions.

Goldstones

The symmetry-breaking pattern is quite involved

$$SU(N_F) \times SU(N_F) \times U(1) \xrightarrow{\text{exp.}} C(M) \times SU(N_F) \xrightarrow{\text{spont.}} C(M).$$

where $C(M) = SU(N_F/2) \times SU(N_F/2) \times U(1)^2$.

Type-I and type-II Goldstones.

- One conformal Goldstone $\omega = \frac{p}{\sqrt{3}}$, which is a singlet of $C(M)$
- One bifundamental with $\omega = \frac{p^2}{2\mu}$
- One field in the $(\mathbf{Adj}, \mathbf{1})$ and one in the $(\mathbf{1}, \mathbf{Adj})$ with $\omega = \sqrt{\frac{\alpha_h}{3\alpha_h + 2\alpha_v}} p$

Total count:

$$1 + 2 \times (N_F/2)^2 + 2 \times (N_F^2/4 - 1) = N_F^2 - 1 = \dim(SU(N_F))$$

What happened to the fermions?

We have only looked at the scalar sector.

The fermions have a large mass that comes from two places:

- The kinetic term, since we have effectively a flat connection
- the Yukawa term $y\,\mathrm{Tr}\left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\right)$ and the vacuum expectation value (VEV) of H

To be precise:

$$m_\psi^2 = \mu^2 + y^2 b^2 \propto \mathcal{J}^{2/3}$$

So they decouple from the dynamics.

The gluons

Now that the fermions have decoupled, since there is no direct connection between the gluons and the scalars, also the gluons decouple.
They will have the usual gap, fixed by the fermion mass

$$\Lambda_{YM} = m_\psi \exp\left[-\frac{3}{22\alpha_g}\right]$$

This will give exponentially small corrections to all the terms in the $\mathcal{J}$ expansion.
In our approximation they can be neglected.

Summing it up

- We can use the large-charge expansion for asymptotically safe theories
- Being in the UV, the large-charge condition is more natural
- For the QCD-inspired model that we have considered:
    - Fermions and gluons decouple.
    - $1/\mathcal{J}$ expansion of the anomalous dimensions, starting at $\mathcal{J}^{4/3}$
    - Rich spectrum of Goldstone modes, with linear and quadratic dispersions.

Going away from conformality

Going away away from conformality

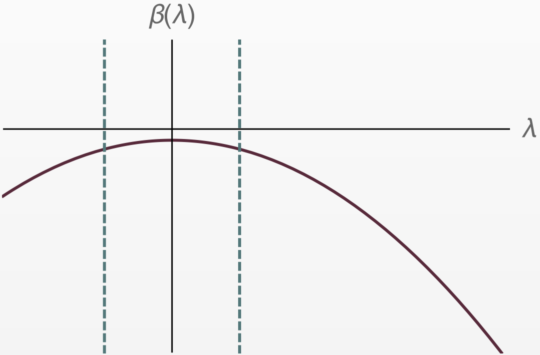
CFTs are very interesting but very constrained.

There is a lot of interesting physics that happens away from conformality.
If we don't go "too far" we can still use large charge effectively.

We will find a very distinct signature of new physics associated to a small dilaton mass in the EFT.

Walking dynamics

For example the walking phase when β functions get close to zero remaining very flat.



The EFT

We mimick it adding a small mass for the dilaton.

Consider a system with $U(1)$ global symmetry in four dimensions.

$$L[\sigma, \chi] = \frac{f_\pi^2 e^{-2f\sigma}}{2} \partial_\mu \chi \, \partial_\mu \chi - e^{-4f\sigma} C^4 + \frac{e^{-2f\sigma}}{2} \left(\partial_\mu \sigma \, \partial_\mu \sigma - \frac{\xi R}{f^2} \right) - \frac{m_\sigma^2}{16 f^2} \left(e^{-4f\sigma} + 4f\sigma - 1 \right)$$

m_σ is the mass of σ (around $\sigma = 0$) that is due to the underlying (walking) dynamics.
It measures the breaking of scale invariance

$$T^\mu{}_\mu = \frac{m_\sigma^2}{f} \sigma .$$

What is the dilaton mass?

In the conformal model at fixed charge the fluctuations of the dilaton around the classical solution are heavy.

Very little to do with m_σ, which is a measure of how much the full theory is non-conformal.

In the large charge approach it will appear in the semiclassical ground state energy.
The semiclassical state resums the quantum effects.

The ground state energy

We just need to solve at fixed values of the charge.

The energy in the cylinder frame has a new, characteristic term

$$r_0 E_{\text{cyl}} = \frac{c_{4/3}}{(4\pi^2)^{1/3}} Q^{4/3} + c_{2/3} Q^{2/3} - \frac{\pi^2 m_\sigma^2 r_0^4}{3f^2} \log(Q) + \dots$$

This is the first time that a $\log(Q)$ term appears in this game.

The two-point function

Close to the fixed point, we can still use the state-operator correspondence.

The two-point function on $\mathbb{R}^4$ for operators of fixed charge is

$$\langle \mathcal{O}_Q(0)\mathcal{O}_{-Q}(x)\rangle = \frac{1}{|x|^{2\Delta}}$$

where Δ has a $\log(Q)$ correction with respect to the dimension at the fixed point Δ^*

$$\Delta = \Delta^* \left(1 - \frac{m_\sigma^2}{24 c_{4/3} f^2 \mu^4} \log(Q)\right)$$

This is a clear signature of a light dilaton in the walking dynamics.

Fluctuations

We can also study the fluctuations on top of the semiclassical fixed-charge state.
We find again two modes.

- A massless mode, which is not anymore exactly conformal

$$\omega = \frac{1}{\sqrt{3}}\left(1 + \frac{m_\sigma^2}{9c_{4/3}f^2\mu^4}\right)p$$

- A massive mode which has essentially the same mass as in the CFT case

$$\omega = 2\mu + \frac{p^2}{2\mu}$$

This is the mass of the fluctuation of σ around the VEV.

Summing it up

- The large-charge approach can be used for walking theories.
- We predict a precise signature of a light dilaton in the two-point functions.
- We have shown the mechanism for the simplest theory.
- The construction can be easily generalized to more realistic situations (around the conformal window).

In conclusion

- With the large-charge approach we can study strongly-coupled systems perturbatively.
- Select a sector and we write a controllable effective theory.
- The strongly-coupled physics is (for the most part) subsumed in a semiclassical state.
- Compute the CFT data.
- Very good agreement with lattice (supersymmetry, large N).
- Works for walking dynamics.
- Precise and testable predictions.